

Unconventional Cosmology

Robert H. Brandenberger

Abstract I review two cosmological paradigms which are alternative to the current inflationary scenario. The first alternative is the “matter bounce”, a non-singular bouncing cosmology with a matter-dominated phase of contraction. The second is an “emergent” scenario, which can be implemented in the context of “string gas cosmology”. I will compare these scenarios with the inflationary one and demonstrate that all three lead to an approximately scale-invariant spectrum of cosmological perturbations.

1 Introduction

1.1 Overview

“Unconventional cosmology” is the title which I was given for my lectures. Based on my interpretation of this title my job is to lecture on alternatives to the current paradigm of early universe cosmology, the “conventional theory”. The fact that almost all cosmologists agree that there is a current paradigm speaks to the remarkable progress of cosmology over the past three decades. At the time when the current paradigm of early universe cosmology, the inflationary scenario [1] (see also [2, 3, 4]), was developed, we had very little observational information about the large-scale structure of the universe. The success of inflationary cosmology at that point in time is that it could explain some of the puzzles which the previous paradigm - Standard Big Bang cosmology - could not address. It was very soon realized [5] (see also [6, 4, 7])) that inflation was much more powerful than simply being able to explain puzzles of Standard Big Bang cosmology such as the flatness

Robert H. Brandenberger
Physics Department, McGill University, 3600 University Street, Montreal, QC H3A 2T8, Canada
e-mail: rhb@physics.mcgill.ca

and horizon problems. In fact, inflationary cosmology gave rise to the first explanation for the origin of inhomogeneities in the universe based on causal physics: It yields a mechanism for generating an approximately scale-invariant spectrum of primordial density fluctuations, i.e. the kind of spectrum which had already been suggested as a reasonable one to be consistent with the (at that time limited) information about the distribution of galaxies [8, 9]. As already realized earlier, such a primordial spectrum of density fluctuations leads to an angular power spectrum of anisotropies in the cosmic microwave background (CMB) radiation which is scale-invariant on large scales and characterized by acoustic oscillations on angular scales of a degree and lower [10, 11]. This prediction has now been confirmed observationally [12] with high accuracy. It is important, however, to keep in mind that any theory which yields an approximately scale-invariant spectrum of primordial fluctuations - and I will present a couple of such theories in these lectures - will agree with the recent high-precision observations of the large-scale structure and CMB anisotropies. Thus, current observations cannot be interpreted as a proof that inflation took place.

In spite of its phenomenological success, inflationary cosmology suffers from some important conceptual problems, which may imply that it is not so “conventional” after all. These problems motivate the search for alternative proposals for the evolution of the early universe and for the generation of structure. These alternatives must be consistent with the current observations, and they must make predictions with which they can be observationally distinguished from inflationary cosmology.

There are indeed paradigms alternative to inflation which generate an almost scale-invariant spectrum of primordial cosmological fluctuations. In these lectures I will present two examples - first the string gas realization [13, 14] (see [15] for reviews) of the emergent universe paradigm [16], and second the “matter bounce scenario” [17, 18] (see [19] for reviews). I should emphasize, however, that these are not the only alternatives to the inflationary scenario. The Pre-Big-Bang scenario [20], the Ekpyrotic scenario [21], and the pseudo-conformal construction [22] are other promising models, and there are others.

The outline of this lecture series is as follows. The first lecture (Sections 1 - 3) focuses on background (homogeneous and isotropic) cosmologies. I begin with a review of the inflationary scenario, the current paradigm of early universe cosmology. After discussing the phenomenological successes of the scenario, I will list a number of conceptual problems which in part motivate the search for alternative scenarios. In Section 2 I introduce the first alternative paradigm which will be discussed here, the “matter bounce” scenario. After presenting the basic idea of the scenario, I will discuss various ways to realize it. In Section 3 I turn to the “emergent Universe” scenario. Once again, I begin by presenting the basic ideas before turning to a discussion of “string gas cosmology”, the specific realization which has provided some very interesting results.

The second lecture (Sections 4 - 7) focuses on the question of how the inhomogeneities and anisotropies which are observed now in the distribution of galaxies on large scales and in temperature maps of the CMB, respectively, are generated. I will

first (Section 4) briefly review the theory of cosmological perturbations. Then, I will emphasize that all three scenarios (inflation, the matter bounce and string gas cosmology), yield fluctuations in agreement with current data, but are distinguishable by future observations. Fluctuations in inflation are reviewed in Section 5, those in the matter bounce in Section 6, and those in string gas cosmology in Section 7. The final section focuses on outstanding problems of the various scenarios, and contains some general discussion.

These lectures are a modified version of lectures given previously [19] at various summer schools.

1.2 Review of Inflationary Cosmology

Inflationary cosmology [1] addresses several shortcomings of Standard Big Bang cosmology (the previous paradigm of early universe cosmology). It explains why the universe is to a good approximation homogeneous and isotropic on large scales (the “horizon problem”), it explains why it is to an excellent accuracy spatially flat (the “flatness problem”), and it can explain its large size and entropy from initial conditions where the universe is of microscopic size.

The idea of inflationary cosmology is to add a period to the evolution of the very early universe during which the scale factor undergoes accelerated expansion - most often nearly exponential growth. To obtain exponential expansion of space in the context of Einstein gravity, the energy density must be constant. Thus, during inflation the total energy and size of the universe both increase exponentially. In this way, inflation can solve the size and entropy problems of Standard Cosmology. Since the horizon expands exponentially during the period of inflation and all classical fluctuations redshift, inflation produces an approximately homogeneous and isotropic space. In addition, the relative contribution of spatial curvature decreases during the period of inflation. Thus, inflation can also address the “flatness problem” of Standard Big Bang cosmology. Any “unconventional cosmology” which claims to provide an alternative to inflation must also address the basic problems of Standard Cosmology mentioned above.

The time line of inflationary cosmology is sketched in Figure 1. The time t_i is the beginning of the inflationary period, and t_R is its end (the meaning of the subscript R will become clear later). Although inflation is usually associated with physics at very high energy scales, e.g. $E \sim 10^{16}$ GeV, all that is required from the initial basic considerations is that inflation ends before the time of nucleosynthesis.

During the period of inflation, the density of any pre-existing particles is diluted exponentially. Hence, if inflation is to be viable, it must contain a mechanism to heat the universe at t_R , a “reheating” mechanism - hence the subscript R on t_R . This mechanism must involve dramatic entropy generation. It is this non-adiabatic evolution which leads to a solution of the flatness problem.

Inflationary cosmology, however, does more than simply solve some conceptual problems of the previous paradigm. It for the first time provided a causal theory of

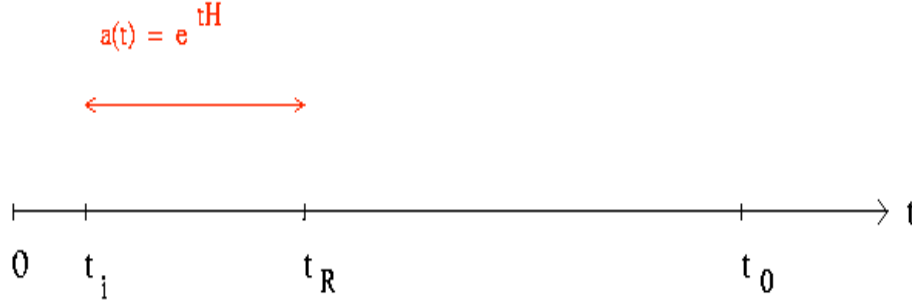


Fig. 1 A sketch showing the time line of inflationary cosmology. The period of accelerated expansion begins at time t_i and end at t_R . The time evolution after t_R corresponds to what happens in Standard Cosmology.

structure formation. Any proposed alternative to cosmological inflation must also match this success. Here we review the basic idea of why inflationary cosmology can provide a causal explanation of the observed inhomogeneities in the universe. The calculations will be reviewed in the second lecture.

In order to understand why inflation provides a causal structure formation mechanism, we start with a space-time sketch of inflationary cosmology as presented in Figure 2. The vertical axis is time, the horizontal axis corresponds to physical distance. Three different distance scales are shown. The solid line labelled by k is the physical length corresponding to a fixed comoving perturbation. The second solid line (blue) is the Hubble radius

$$l_H(t) \equiv H^{-1}(t). \quad (1)$$

As will be shown in Lecture 2, the Hubble radius separates scales where microphysics dominates over gravity (sub-Hubble scales) from ones on which the effects of microphysics are negligible (super-Hubble scales). Hence, a necessary requirement for a causal theory of structure formation is that scales we observe today originate at sub-Hubble lengths in the early universe. The third length is the “horizon”, the forward light cone of our position at the Big Bang. The horizon is the causality limit. Note that because of the exponential expansion of space during inflation, the horizon is exponentially larger than the Hubble radius. It is important not to confuse these two scales. Hubble radius and horizon are the same in Standard Cosmology, but in all three early universe scenarios which will be discussed in these lectures they are completely different (in inflationary cosmology the horizon is exponentially larger, in the matter bounce scenario it is in fact infinite, and in the emergent scenario it is infinite if the emergent phase extends to $t = -\infty$). In fact, in any structure formation scenario the two scales need to be different.

From Fig. 2 it is clear that provided the period of inflation is sufficiently long, all scales which are currently observed originate as sub-Hubble scales at the beginning of the inflationary phase. Thus, in inflationary cosmology it is possible to have a causal generation mechanism of fluctuations [5, 6, 4]. Since matter pre-existing at

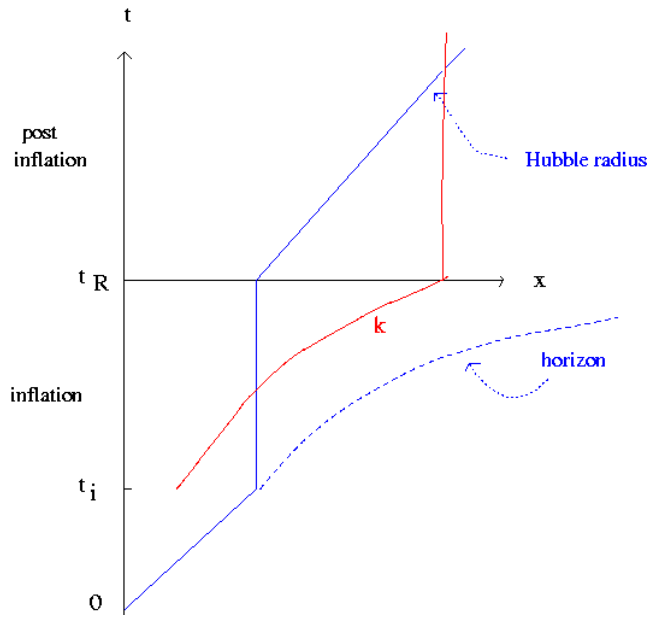


Fig. 2 Space-time sketch of inflationary cosmology. The vertical axis is time, the horizontal axis corresponds to physical distance. The solid line labelled k is the physical length of a fixed comoving fluctuation scale. The role of the Hubble radius and the horizon are discussed in the text.

t_i is redshifted away, we are left with a matter vacuum. The inflationary universe scenario of structure formation is based on the hypothesis that all current structure originated as quantum vacuum fluctuations. From Figure 2 it is also clear that the horizon problem of standard cosmology can be solved provided that the period of inflation lasts sufficiently long. For inflation to solve the horizon and flatness problem of Standard cosmology, the period of exponential expansion must be greater

than about $50H^{-1}$ (this number depends very slightly on the energy scale at which inflation takes place).

In order to obtain exponential expansion of space in the context of Einstein gravity, matter with an equation of state $p = -\rho$ is required, where p and ρ are pressure and energy density, respectively. In the context of renormalizable quantum field theory, a phase dominated by almost constant (both in space and time) potential energy of a scalar matter field is required.

1.3 Conceptual Problems of Inflationary Cosmology

In spite of the phenomenological success of the inflationary paradigm, conventional scalar field-driven inflation suffers from several important conceptual problems.

The first problem concerns the nature of the inflaton, the scalar field which generates the inflationary expansion. No particle corresponding to the excitation of a scalar field has yet been observed in nature, and the Higgs field which is introduced to give elementary particles masses in the Standard Model of particle physics does not have the required flatness of the potential to yield inflation, unless it is non-minimally coupled to gravity [23]. In particle physics theories beyond the Standard Model there are often many scalar fields, but it is in general very hard to obtain the required flatness properties on the potential.

The second problem (the **amplitude problem**) relates to the amplitude of the spectrum of cosmological perturbations. In a wide class of inflationary models, obtaining the correct amplitude requires the introduction of a hierarchy in scales, namely [24]

$$\frac{V(\varphi)}{\Delta\varphi^4} \leq 10^{-12}, \quad (2)$$

where $\Delta\varphi$ is the change in the inflaton field during the minimal length of the inflationary period, and $V(\varphi)$ is the potential energy during inflation.

A more serious problem is the **trans-Planckian problem** [25]. Returning to the space-time diagram of inflation (see Figure 3), we can immediately deduce that, provided that the period of inflation lasted sufficiently long (for GUT scale inflation the number is about 70 e-foldings), then all scales inside the Hubble radius today started out with a physical wavelength smaller than the Planck scale at the beginning of inflation. Now, the theory of cosmological perturbations is based on Einstein's theory of General Relativity coupled to a simple semi-classical description of matter. It is clear that these building blocks of the theory are inapplicable on scales comparable and smaller than the Planck scale. Thus, the key successful prediction of inflation (the theory of the origin of fluctuations) is based on suspect calculations since new physics *must* enter into a correct computation of the spectrum of cosmological perturbations. The key question is as to whether the predictions obtained using the current theory are sensitive to the specifics of the unknown theory which takes over on small scales. Simple toy models of new physics on super-Planck scales based on modified dispersion relations were used in [26] (see also [27]) to show that the

resulting spectrum of cosmological fluctuations indeed depends on what is assumed about physics on trans-Planckian scales.

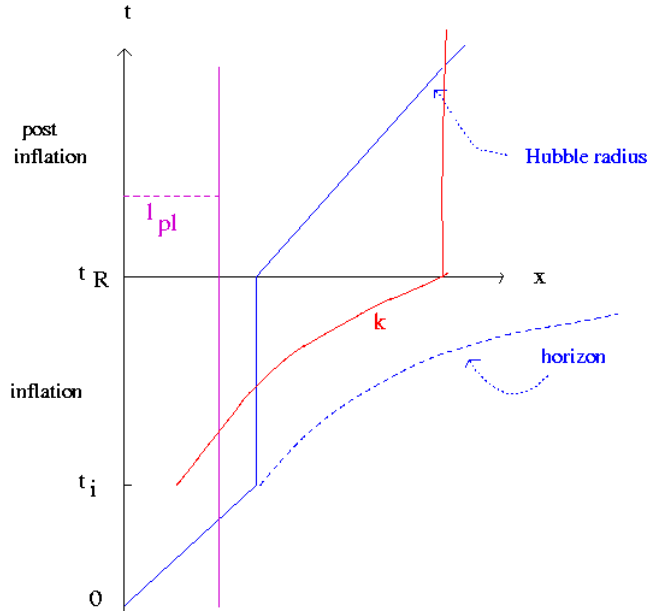


Fig. 3 Space-time diagram (sketch) of inflationary cosmology where we have added an extra length scale, namely the Planck length l_{pl} (magenta vertical line). The symbols have the same meaning as in Figure 2. Note, specifically, that - as long as the period of inflation lasts a couple of e-foldings longer than the minimal value required for inflation to address the problems of Standard Big Bang cosmology - all wavelengths of cosmological interest to us today start out at the beginning of the period of inflation with a wavelength which is smaller than the Planck length.

A fourth problem is the **singularity problem**. It was known for a long time that Standard Big Bang cosmology cannot be the complete story of the early universe because of the initial singularity, a singularity which is unavoidable when basing cosmology on Einstein's field equations in the presence of a matter source obeying the weak energy conditions (see e.g. [28] for a textbook discussion). The singularity theorems have been generalized to apply to Einstein gravity coupled to scalar field matter, i.e. to scalar field-driven inflationary cosmology [29]. It was shown that, in this context, a past singularity at some point in space is unavoidable. Thus we know, from the outset, that scalar field-driven inflation cannot be the ultimate theory of the very early universe.

The Achilles heel of scalar field-driven inflationary cosmology may be the **cosmological constant problem**. We know from observations that the large quantum vacuum energy of field theories does not gravitate today. However, to obtain a period of inflation one is using the part of the energy-momentum tensor of the scalar field which looks like the vacuum energy. In the absence of a solution of the cosmological constant problem it is unclear whether scalar field-driven inflation is robust, i.e. whether the mechanism which renders the quantum vacuum energy gravitationally inert today will not also prevent the vacuum energy from gravitating during the period of slow-rolling of the inflaton field.

A final problem which we will mention here is the concern that the energy scale at which inflation takes place is too high to justify an effective field theory analysis based on Einstein gravity. In simple toy models of inflation, the energy scale during the period of inflation is about 10^{16}GeV , very close to the string scale in many string models, and not too far from the Planck scale. Thus, correction terms in the effective action for matter and gravity may already be important at the energy scale of inflation, and the cosmological dynamics may be rather different from what is obtained when neglecting the correction terms.

In Figure 4 we show once again the space-time sketch of inflationary cosmology. In addition to the length scales which appear in the previous versions of this figure, we have now shaded the “zones of ignorance”, zones where the Einstein gravity effective action is sure to break down. As described above, fluctuations emerge from the short distance zone of ignorance (except if the period of inflation is very short), and the energy scale of inflation might put the period of inflation too close to the high energy density zone of ignorance to trust the predictions based on using the Einstein action.

The arguments in this subsection provide a motivation for considering alternative scenarios of early universe cosmology. Below we will focus on two scenarios, the *matter bounce* and *string gas cosmology*, a realization of the *emergent universe* paradigm.

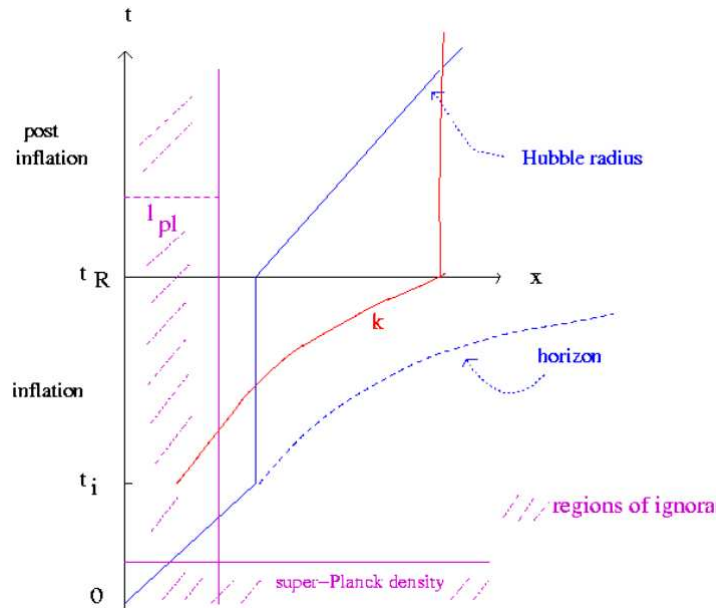


Fig. 4 Space-time diagram (sketch) of inflationary cosmology including the two zones of ignorance - sub-Planckian wavelengths and trans-Planckian densities. The symbols have the same meaning as in Figure 2. Note, specifically, that - as long as the period of inflation lasts a couple of e-foldings longer than the minimal value required for inflation to address the problems of Standard Big Bang cosmology - all wavelengths of cosmological interest to us today start out at the beginning of the period of inflation with a wavelength which is in the zone of ignorance.

2 Matter Bounce

2.1 The Idea

The first alternative to cosmological inflation as a theory of structure formation is the “matter bounce”, an alternative which is not yet well appreciated (for an overview the reader is referred to [19]). The scenario is based on a cosmological background in which the scale factor $a(t)$ bounces in a non-singular manner.

Figure 5 shows a space-time sketch of such a bouncing cosmology. Without loss of generality we can adjust the time axis such that the bounce point (minimal value of the scale factor) occurs at $t = 0$. There are three phases in such a non-singular bounce: the initial contracting phase during which the Hubble radius is decreasing linearly in $|t|$, a bounce phase during which a transition from contraction to expansion takes place, and thirdly the usual expanding phase of Standard Cosmology. There is no prolonged inflationary phase after the bounce, nor is there a time-symmetric deflationary contracting period before the bounce point. As is obvious from the Figure, scales which we observe today started out early in the contracting phase at sub-Hubble lengths. The matter bounce scenario assumes that the contracting phase is matter-dominated at the times when scales we observe today exit the Hubble radius. A model in which the contracting phase is the time reverse of our current expanding phase would obey this condition. The assumption of an initial matter-dominated phase will be seen later in Lecture 2 to be important if we want to obtain a scale-invariant spectrum of cosmological perturbations from initial vacuum fluctuations [17, 18].

Let us make a first comparison with inflation. A non-deflationary contracting phase replaces the accelerated expanding phase as a mechanism to bring fixed comoving scales within the Hubble radius as we go back in time, allowing us to consider the possibility of a causal generation mechanism of fluctuations. Starting with vacuum fluctuations, a matter-dominated contracting phase is required in order to obtain a scale-invariant spectrum. This corresponds to the requirement in inflationary cosmology that the accelerated expansion be nearly exponential.

How are the problems of Standard Big Bang cosmology addressed in the matter bounce scenario? First of all, note that since the universe begins cold and large, the size and entropy problems of Standard Cosmology do not arise. As is obvious from Figure 5, there is no horizon problem for the matter bounce scenario as long as the contracting period is long (to be specific, of similar duration as the post-bounce expanding phase until the present time). By the same argument, it is possible to have a causal mechanism for generating the primordial cosmological perturbations which evolve into the structures we observe today. Specifically, as will be discussed in Section 6, if the fluctuations originate as vacuum perturbations on sub-Hubble scales in the contracting phase, then the resulting spectrum at late times for scales exiting the Hubble radius in the matter-dominated phase of contraction is scale-invariant [17, 18].

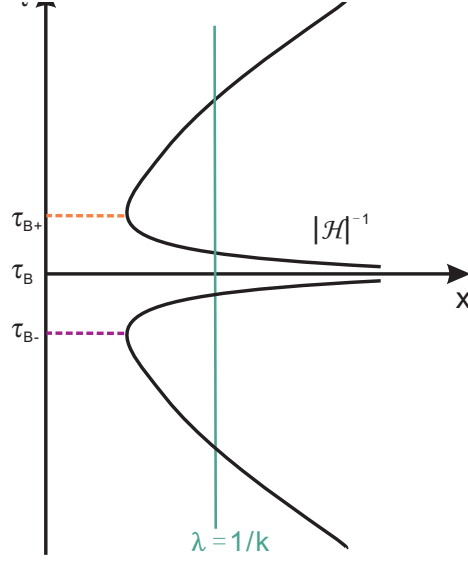


Fig. 5 Space-time sketch in the matter bounce scenario. The vertical axis is conformal time η , the horizontal axis denotes a co-moving space coordinate. The vertical line indicates the wavelength of a fluctuation mode. Also, \mathcal{H}^{-1} denotes the co-moving Hubble radius.

The flatness problem is the one which is only partially addressed in the matter bounce setup. The contribution of the spatial curvature decreases in the contracting phase at the same rate as it increases in the expanding phase. Thus, to explain the observed spatial flatness, comparable spatial flatness at early times in the contracting phase is required. This is an improved situation compared to the situation in Standard Big Bang cosmology where spatial flatness is overall an unstable fixed point and hence extreme fine tuning of the initial conditions is required to explain the observed degree of flatness. But the situation is not as good as it is in a model with a long period of inflation where spatial flatness is a local attractor in initial condition space (it is not a global attractor, though!).

How does the matter bounce scenario address the conceptual problem of inflation? First of all, the length scale of fluctuations of interest for current observations on cosmological scales is many orders of magnitude larger than the Planck length throughout the evolution. If the energy scale at the bounce point is comparable to the particle physics GUT scale, then typical wavelengths at the bounce point are not too different from 1mm. Hence, the fluctuations never get close to the small wavelength zone of ignorance in Figures 3 and 4, and thus a description of the evolution of fluctuations using Einstein gravity should be well justified modulo possible dif-

difficulties at the bounce point which we will return to in Section 6. Thus, there is no trans-Planckian problem for fluctuations in the matter bounce scenario.

As will be discussed below, new physics is required in order to provide a non-singular bounce. Thus, the “solution” of the singularity problem is put in by hand and cannot be counted as a success, except in realizations of the matter bounce in the context of a string theory background in which the non-singular evolution follows from general principles. Such a theory has recently been presented in [30] (see [31] for an analysis of fluctuations in these models). Existing matter bounce models do not address the cosmological constant problem. However, I would like to emphasize that the mechanism which drives the evolution in the matter bounce scenario is robust against our ignorance of what solves the cosmological constant problem, an improvement of the situation compared to the situation in inflationary cosmology.

With Einstein gravity and matter satisfying the usual energy conditions it is not possible to obtain a non-singular bounce. Thus, new physics is required in order to obtain a non-singular bouncing cosmology. Such new physics can arise by modifying the gravitational sector, or by modifying the matter sector. The study of bouncing cosmologies has a long history (see [32] for an in-depth review of a lot of these past approaches). We will now turn to a brief overview of some more recent work on non-singular bouncing cosmology with the matter bounce in mind.

2.2 Realizing a Matter Bounce with Modified Matter

In order to obtain a cosmological bounce in the context of Einstein gravity, it is necessary to introduce a new form of matter which violates the Null Energy Condition (NEC). A simple way to do this is by introducing quintom matter [33]. Resulting nonsingular quintom bouncing models have been discussed in [34]. Quintom matter is a set of two matter fields, one of them regular matter (obeying the NEC), the second a “phantom” field with opposite sign kinetic term which violates the NEC. Even though this model is plagued by ghost instabilities [35], we will use it to illustrate the basic idea of how a bouncing cosmology can be obtained.

We [34] model both matter components with scalar fields, the mass of the regular one (ϕ) being m , and M being that of the field $\tilde{\phi}$ with wrong sign kinetic term. We consider a contracting universe and assume that early on both fields are oscillating, but that the amplitude \mathcal{A} of ϕ greatly exceeds the corresponding amplitude $\tilde{\mathcal{A}}$ of $\tilde{\phi}$ such that the energy density is dominated by ϕ . During the initial period of contraction, both amplitudes grow at the same rate. At some point, \mathcal{A} will become so large that the oscillations of ϕ freeze out¹. Then, \mathcal{A} will grow only slowly, whereas $\tilde{\mathcal{A}}$ will continue to increase. Thus, the (negative) energy density in $\tilde{\phi}$ will grow in absolute value relative to that of ϕ . The total energy density will decrease towards 0. At that point, $H = 0$ by the Friedmann equations. Since it is only the phantom

¹ This corresponds to the time reverse of entering a region of large-field inflation.

field which has large kinetic energy, it follows that $\dot{H} > 0$ when $H = 0$. Hence, a non-singular bounce occurs.

The Higgs sector of the Lee-Wick model [36] provides a concrete realization of the quintom bounce model, as studied in [37]. Quintom models like all other models with negative sign kinetic terms suffer from an instability problem [35] in the matter sector and are hence problematic. In addition, they are unstable against the addition of radiation (see e.g. [38]) and anisotropic stress (the BKL instability [39]).

An improved way of obtaining a non-singular bouncing cosmology using modified matter is by using a ghost condensate field [40] (see also [41, 42] where ghost condensates have been used to produce non-singular bounces in different contexts). The ghost condensation mechanism is the analog of the Higgs mechanism in the kinetic sector of the theory. In the Higgs mechanism we take a field ϕ whose mass when evaluated at $\phi = 0$ is tachyonic, add higher powers of ϕ^2 to the potential term in the Lagrangian such that there is a stable fixed point $\phi = v \neq 0$, and thus when expanded about $\phi = v$ the mass term has the “safe” non-tachyonic sign. In the ghost condensate construction we take a field ϕ whose kinetic term

$$X \equiv -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \quad (3)$$

appears with the wrong sign in the Lagrangian. Then, we add higher powers of X to the kinetic Lagrangian such that there is a stable fixed point $X = c^2$ and such that when expanded about $X = c^2$ the fluctuations have the regular sign of the kinetic term:

$$\mathcal{L} = \frac{1}{8} M^4 (X - c^2)^2 - V(\phi), \quad (4)$$

where $V(\phi)$ is a usual potential function, M is a characteristic mass scale and the dimensions of ϕ are chosen such that X is dimensionless.

In the context of cosmology, the ghost condensate is

$$\phi = ct \quad (5)$$

and breaks local Lorentz invariance. Now let us expand the homogeneous component of ϕ about the ghost condensate:

$$\phi(t) = ct + \pi(t). \quad (6)$$

If $\dot{\pi} < 0$ then the gravitational energy density is negative, and a non-singular bounce is possible. Thus, in [40] we constructed a model in which the ghost condensate field starts at negative values and the potential $V(\phi)$ is negligible. As ϕ approaches $\phi = 0$ it encounters a positive potential which slows it down, leading to $\dot{\pi} < 0$ and hence to negative gravitational energy density. Thus, a non-singular bounce can occur. We take the potential to be of the form

$$V(\phi) \sim \phi^{-\alpha} \quad (7)$$

for $|\phi| \gg M$, where M is the mass scale above which the higher derivative kinetic terms are important. For sufficiently large values of α , namely

$$\alpha \geq 6, \quad (8)$$

the energy density in the ghost condensate increases faster than that of radiation and anisotropic stress at the universe contracts. Hence, this bouncing cosmology is locally stable against the addition of radiation and anisotropic stress (there is still an instability to the development of anisotropic stress in the contracting phase prior to the time when the ghost condensate starts to dominate).

Non-singular bouncing cosmologies can also be obtained making use of Galileon models [43]. However, these models also suffer from an instability against the development of anisotropic stress.

The Ekpyrotic contracting universe (contracting phase with an equation of state $w \gg 1$ is stable against the growth of anisotropies, as shown in [44]). Thus, one way of obtaining a matter bounce which is stable against the development of anisotropic stress is to have a phase of Ekpyrotic contraction set in shortly after the time t_{eq}^- of equal matter and radiation in the contracting phase. A model in which this is realized and in which the non-singular bounce is generated by a ghost condensate and Galileon construction has recently been worked out in [45].

2.3 Realizing a Matter Bounce with Modified Gravity

It is unreasonable to expect that Einstein gravity will provide a good description of the physics at very high energy densities. In particular, all approaches to quantum gravity lead to correction terms in the gravitational action (compared to the pure Einstein term) which become dominant at the Planck scale. It is possible (and in some approaches to quantum gravity such as string theory even likely) that the new terms will tend to prevent cosmological singularities from appearing, and hence might allow a bouncing cosmology even in the presence of matter which obeys the NEC.

One early study is based on a higher derivative Lagrangian resulting from the “nonsingular universe construction” of [46] which is based on a Lagrange multiplier construction which forces all space-time curvature invariants to stay bounded as the energy density increases. This Lagrangian admits bouncing solutions in the presence of regular matter. Another model is the non-local higher derivative action of [47] which is constructed to be ghost-free about Minkowski space-time and which admits bouncing solutions. Mirage cosmology [48] (induced gravity on a brane which is moving into and out of a high-curvature throat of a higher-dimensional bulk space-time also admits bouncing cosmologies [49].

A few years ago there was a lot of interest in Horava-Lifshitz gravity [50], an approach to quantum gravity in four space-time dimensions which is based on a gravitational Lagrangian which is power-counting renormalizable with respect to

the reduced symmetry group of spatial diffeomorphisms only (we drop the invariance requirement under space-dependent time reparametrizations), The lost symmetry is replaced by an anisotropic scaling symmetry between space and time. The Lagrangian contains higher space derivative terms. As was realized in [51], in the presence of spatial curvature these higher space derivative terms act as ghost radiation and ghost anisotropic stress and lead to the possibility of a non-singular bouncing cosmology.

Loop quantum cosmology is an approach to quantum cosmology which also leads to bouncing solutions (see e.g. [52] for a review). What is responsible here for singularity avoidance is the fundamental discreteness of the area which comes from quantization. Other lecturers at this school have discussed loop quantum cosmology in depth.

Superstring theory as a quantum theory which includes gravity will likely also resolve cosmological singularities. As will be discussed in detail in the section on string gas cosmology, the new degrees of freedom which string theory admits compared to point particle theories lead to duality symmetries which relate large and small spaces. Physical quantities such as the temperature remain bounded, and it is hence likely to obtain bouncing cosmological solutions. Our understanding of string cosmology is hampered by the lack of a fully non-perturbative formulation of string theory in a cosmological space-time. Most analyses of string cosmology are performed using string-motivated field theory. A specific theory in which the field theory approximations are under good control is the Type II string cosmology of [30].

3 Emergent Universe

3.1 *The Idea*

The “emergent universe” scenario [16] is another non-singular cosmological scenario in which time runs from $-\infty$ to $+\infty$. The idea is that if we follow the evolution of our homogeneous and isotropic space-time into the past, the expansion rate H ceases to increase as we approach a certain limiting scale (most likely related to the Planck energy). Instead of further increasing, H decreases to zero, and the scale factor approaches a constant value as we tend to past infinity. The time evolution of the scale factor is sketched in Figure (6).

In Figure 7 we sketch the space-time diagram in emergent cosmology. Since the early emergent phase is quasi-static, the Hubble radius is infinite. For the same reason, the physical wavelength of fluctuations remains constant in this phase. At the end of the emergent phase, the Hubble radius decreases to a microscopic value and makes a transition to its evolution in Standard Cosmology.

Once again, we see that fluctuations originate on sub-Hubble scales. In emergent cosmology, it is the existence of a quasi-static phase which leads to this result. What

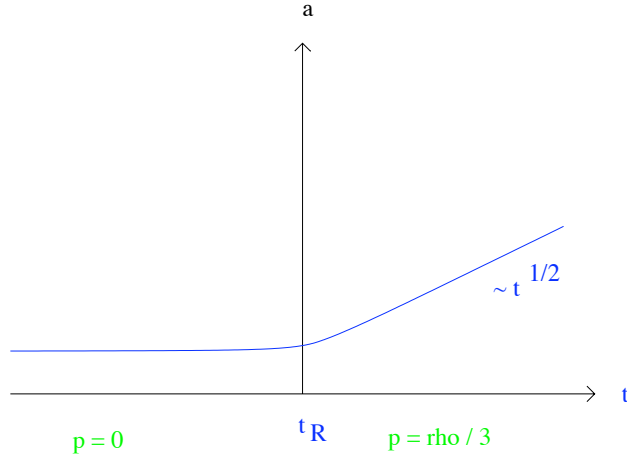


Fig. 6 The dynamics of emergent universe cosmology. The vertical axis represents the scale factor of the universe, the horizontal axis is time.

sources fluctuations depends on the realization of the emergent scenario. String gas cosmology is the example which I will consider later on. In this case, the source of perturbations is thermal: string thermodynamical fluctuations in a compact space with stable winding modes, and this in fact leads to a scale-invariant spectrum [14].

How does emergent cosmology address the problems of Standard Cosmology? As in the case of a bouncing cosmology, the horizon is infinite and hence there is no horizon problem. Since there is likely thermal equilibrium in the emergent phase, a mechanism to homogenize the universe exists, and hence spatial flatness is not a mystery. As discussed in the previous paragraph, there is no causality obstacle against producing cosmological fluctuations. The scenario is non-singular, but this cannot in general be weighted as a success unless the emergent phase can be shown to arise from some well controlled ultraviolet physics.

Like in the case of a bouncing cosmology, there is no trans-Planckian problem for fluctuations - their wavelength never gets close to the Planck scale. And like in the case of a bouncing cosmology, the physics driving the background dynamics is robust against our ignorance of what solves the cosmological constant problem. These are two advantages of the emergent scenario compared to inflation.

On the negative side, the origin of the large size and entropy of our universe remains a mystery in emergent cosmology. Also, the physics yielding the emergent phase is not well understood in terms of an effective field theory setting, in contrast to the physics yielding inflation.

Whereas there are a lot of toy models for a bouncing cosmology, there are not many models that realize an emergent universe. The “String Gas Cosmology” model discussed below is a concrete proposal. Another recent proposal is in the context of Galileon cosmology [53] (see [54] for a discussion of the termination of the emergent phase in the context of the model of [53]). There is also a relationship with the work of [22]. The small number of concrete models, however, does not mean

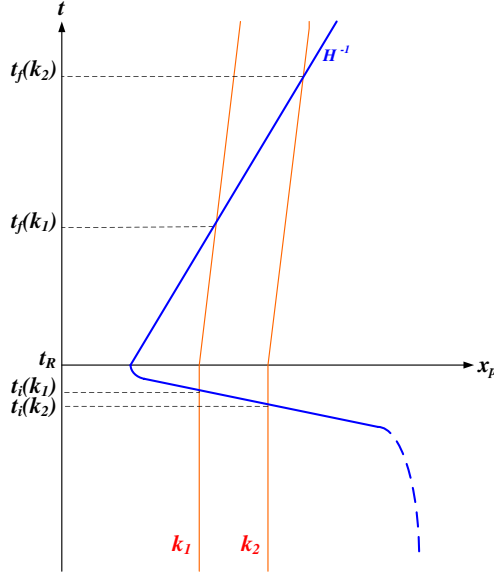


Fig. 7 Space-time diagram (sketch) showing the evolution of fixed co-moving scales in emergent cosmology. The vertical axis is time, the horizontal axis is physical distance. The solid curve represents the Einstein frame Hubble radius H^{-1} which shrinks abruptly to a micro-physical scale at t_R and then increases linearly in time for $t > t_R$. Fixed co-moving scales (the dotted lines labeled by k_1 and k_2) which are currently probed in cosmological observations have wavelengths which were smaller than the Hubble radius long before t_R . They exit the Hubble radius at times $t_i(k)$ just prior to t_R , and propagate with a wavelength larger than the Hubble radius until they re-enter the Hubble radius at times $t_f(k)$.

that this approach is not promising. I suspect that any non-perturbative approach to quantum gravity which leads to an emergence of space after some phase transition will lead to a convincing realization of emergent cosmology.

3.2 String Gas Cosmology

String gas cosmology [13] (see also [55], and see [15] for a review) is a realization of the emergent cosmology paradigm which results from coupling a gas of fundamental strings to a background space-time metric. It is assumed that the spatial sections are

compact. For simplicity, we take all spatial directions to be toroidal and denote the radius of the torus by R .

The guiding principle of string gas cosmology is to focus on symmetries and degrees of freedom which are new to string theory (compared to point particle theories) and which will be part of any non-perturbative string theory, and to use them to develop a new cosmology. The symmetry we make use of is **T-duality**, and the new degrees of freedom are the **string oscillatory modes** (corresponding to fluctuations in the shape of a string) and the **string winding modes** (strings winding the background space). Strings also have **momentum modes** which correspond to the center of mass motion of the strings. Point particles only have momentum modes.

The first key feature of string theory is that there is a limiting temperature for a gas of strings in thermal equilibrium, the *Hagedorn temperature* [56] T_H . This stems from the fact that the number of string oscillatory states increases exponentially with energy. Thus, if we take a box of strings and adiabatically decrease the box size, the temperature will never diverge. This is the first indication that string theory has the potential to resolve the cosmological singularity problem.

The second key feature of string theory upon which string gas cosmology is based is *T-duality*. To introduce this symmetry, let us discuss the radius dependence of the energy of the basic string states: The energy of an oscillatory mode is independent of R , momentum mode energies are quantized in units of $1/R$, i.e.

$$E_n = n\mu \frac{l_s^2}{R}, \quad (9)$$

where l_s is the string length and μ is the mass per unit length of a string. The winding mode energies are quantized in units of R , i.e.

$$E_m = m\mu R, \quad (10)$$

where both n and m are integers. Thus, a new symmetry of the spectrum of string states emerges: Under the change

$$R \rightarrow 1/R \quad (11)$$

in the radius of the torus (in units of l_s) the energy spectrum of string states is invariant if winding and momentum quantum numbers are interchanged

$$(n, m) \rightarrow (m, n). \quad (12)$$

The above symmetry is the simplest element of a larger symmetry group, the T-duality symmetry group which in general also mixes fluxes and geometry. The string vertex operators are consistent with this symmetry, and thus T-duality is a symmetry of perturbative string theory. Postulating that T-duality extends to non-perturbative string theory leads [57] to the need of adding D-branes to the list of fundamental objects in string theory. With this addition, T-duality is expected to be a symmetry of non-perturbative string theory. Specifically, T-duality will take a spectrum of stable

Type IIA branes and map it into a corresponding spectrum of stable Type IIB branes with identical masses [58].

As discussed in [13], the above T-duality symmetry leads to an equivalence between small and large spaces, an equivalence elaborated on further in [59, 60].

Let us now turn to the background cosmology which emerges from string gas cosmology. First consider the adiabatic evolution of a box of strings as the box radius R decreases. If the initial radius is much larger than the string length, then in thermal equilibrium most of the energy is initially in the momentum modes since they are the lightest ones. As R decreases, the temperature first rises as in standard cosmology since the string states which are occupied (the momentum modes) get heavier. However, as the temperature approaches the Hagedorn temperature, the energy begins to flow into the oscillatory modes and the temperature levels off. As the radius R decreases below the string scale, the temperature begins to decrease as the energy begins to flow into the winding modes whose energy decreases as R decreases (see Figure 8). Thus, as argued in [13], the temperature singularity of early universe cosmology is resolved in string gas cosmology.

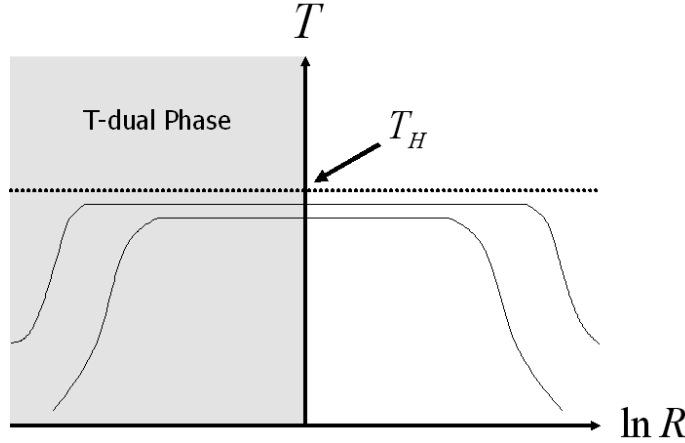


Fig. 8 The temperature (vertical axis) as a function of radius (horizontal axis) of a gas of closed strings in thermal equilibrium. Note the absence of a temperature singularity. The range of values of R for which the temperature is close to the Hagedorn temperature T_H depends on the total entropy of the universe. The upper of the two curves corresponds to a universe with larger entropy.

The equations that govern the background of string gas cosmology are not known. The Einstein equations are not the correct equations since they do not obey the T-duality symmetry of string theory. Many early studies of string gas cosmology were based on using the dilaton gravity equations [61, 62, 63]. However, these equations are not satisfactory, either. Firstly, we expect that string theory correction terms to the low energy effective action of string theory become dominant in the Hagedorn phase. Secondly, the dilaton gravity equations yield a rapidly changing dilaton during the Hagedorn phase (in the string frame). Once the dilaton becomes

large, it becomes inconsistent to focus on fundamental string states rather than brane states. In other words, using dilaton gravity as a background for string gas cosmology does not correctly reflect the S-duality symmetry of string theory. A background for string gas cosmology including a rolling tachyon was proposed [64] which allows a background in the Hagedorn phase with constant scale factor and constant dilaton; but this construction is rather ad hoc. Another study of this problem was given in [65].

Some conclusions about the time-temperature relation in string gas cosmology can be derived based on thermodynamical considerations alone. One possibility is that R starts out much smaller than the self-dual value and increases monotonically. From Figure 8 it then follows that the time-temperature curve will correspond to that of a bouncing cosmology. A specific realization of this possibility in the context of a string theory background in which the effective background equations of motion are well justified is given in [30].

Alternatively, it is possible that the universe starts out in a meta-stable state near the Hagedorn temperature, the *Hagedorn phase*, and then smoothly evolves into an expanding phase dominated by radiation like in Standard Cosmology. Note that we are assuming that not only the scale factor but also the dilaton is constant in time. This is the setup which is assumed in the string gas realization of the emergent universe scenario.

Note that it is the annihilation (see Figure (9)) of winding strings into string loops (which acts as stringy radiation) which leads to the transition from the early quasi-static phase to the radiation phase of Standard Cosmology.

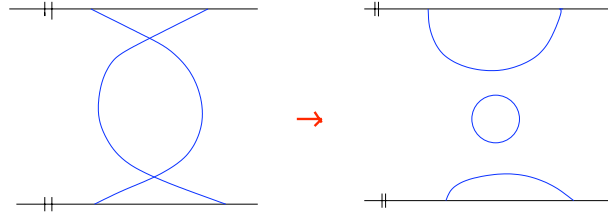


Fig. 9 The process by which string loops are produced via the intersection of winding strings. The top and bottom lines are identified and the space between these lines represents space with one toroidal dimension un-wrapped.

The evolution of the scale factor in string gas cosmology is as in any general emergent universe scenario (see Fig. 6). In this figure, along the horizontal axis, the approximate equation of state for the string gas cosmology realization of the emergent scenario is also indicated. During the Hagedorn phase the pressure is negligible due to the cancellation between the positive pressure of the momentum modes and the negative pressure of the winding modes, after time t_R the equation of state is that of a radiation-dominated universe.

As pointed out in [13], the annihilation process which allows for the expansion of spatial radii is only possible in at most three large spatial dimensions. This is a

simple dimension counting argument: string world sheets have measure zero intersection probability in more than four large space-time dimensions. Hence, string gas cosmology may provide a natural mechanism for explaining why there are exactly three large spatial dimensions. This argument was supported by numerical studies of string evolution in three and four spatial dimensions [66] (see also [67]). The flow of energy from winding modes to string loops can be modelled by effective Boltzmann equations [68] analogous to those used to describe the flow of energy between infinite cosmic strings and cosmic string loops (see e.g. [69, 70, 71] for reviews).

There is a caveat regarding the above mechanism. In the analysis of [68] it was assumed that the string interaction rates were time-independent. If the dynamics of the Hagedorn phase is modelled by dilaton gravity, the dilaton is rapidly changing during the phase in which the string frame scale factor is static. As discussed in [72, 73] (see also [74]), in this case the mechanism which tells us that exactly three spatial dimensions become macroscopic does not work.

An important question which has to be addressed in any model of string cosmology is what stabilizes the moduli, in particular the sizes and shapes of the extra spatial dimensions. In this respect string gas cosmology in the context of heterotic string theory has some major advantages over other approaches to string cosmology, at least in the context of toroidal compactifications, the ones which have been studied to date. This issue is reviewed in detail in [75]. The basic idea [76] is that winding modes about the extra spatial dimensions create an energy barrier against expansion, whereas momentum modes cause an energy barrier against contraction. There is hence an energetically favored value for the radius R of an extra spatial dimension (which is typically the string length). This is the self-dual radius. This mechanism is a special case of the general principle of moduli trapping at enhanced symmetry states [77, 78].

In order to avoid a cosmological constant problem, it is important that the induced potential energy of the four-dimensional effective field theory vanishes at the self-dual radius. This issue has been studied in detail [79, 80] in the case of heterotic superstring theory, and it was shown that the special massless enhanced symmetry states which appear at the self-dual radius and dominate the potential at that point have vanishing potential energy. Thus, in heterotic string gas cosmology the radion moduli are dynamically stabilized. By studying the off-diagonal Einstein equations in the presence of a string gas with both momentum and winding modes it can also be shown [81] that the shape moduli are stabilized at points of extra symmetry.

The only modulus which is not stabilized by string winding and momentum modes is the dilaton. One can [82] introduce gaugino condensation, the same mechanism used in string inflation model building (see e.g. [83] for a recent review) and show that this generates a stabilizing potential for the dilaton without interfering with the radion stabilization force provided by the string winding and momentum modes. Gaugino condensation also leads [84] to supersymmetry breaking (typically at a high energy scale).

A final comment concerns the isotropy of the three large dimensions. In contrast to the situation in Standard cosmology, in string gas cosmology the anisotropy

decreases in the expanding phase [85]. Thus, there is a natural isotropization mechanism for the three large spatial dimensions.

4 Cosmological Perturbations

4.1 Overview

The topic of the second lecture is the theory of cosmological perturbations and its applications to both inflationary cosmology and the “un-conventional” cosmological alternatives discussed in the first lecture.

The theory of cosmological perturbations is the main tool of modern cosmology. It allows us to follow the evolution of small inhomogeneities generated in the very early universe and propagate their evolution to the present time, which then allows us to work out predictions of models of the early universe. For an extensive overview of the subject the reader is referred to [86], and to [87] for an overview.

As we have seen in the first lecture, in many models of the very early Universe, in particular in inflationary cosmology, in the emergent universe paradigm and in the matter bounce scenario, primordial inhomogeneities are generated in an initial phase on sub-Hubble scales. The wavelength is then stretched relative to the Hubble radius $H^{-1}(t)$, where H is the cosmological expansion rate, becomes larger than the Hubble radius at some time and then propagates on super-Hubble scales until re-entering at late cosmological times. In a majority of the current structure formation scenarios (string gas cosmology is an exception in this respect), fluctuations are assumed to emerge as quantum vacuum perturbations. Hence, to describe the generation and evolution of the inhomogeneities, both General Relativity and quantum mechanics are required. What makes the theory of cosmological perturbations tractable is that the amplitude of the fractional fluctuations is small today and hence (since gravity is a purely attractive force) that it was even smaller in the early universe. This justifies the linear analysis of the generation and evolution of fluctuations.

In the context of a Universe with an inflationary period, the quantum origin of cosmological fluctuations was first discussed in [5] and also [6, 4] for earlier ideas. In particular, Mukhanov [5] and Press [6] realized that in an exponentially expanding background, the curvature fluctuations would be scale-invariant, and Mukhanov provided a quantitative calculation which also yielded the logarithmic deviation from exact scale-invariance.

Here we give a very abbreviated overview of the quantum theory of cosmological perturbations. The reader is referred to [87] for a description which is closer to what was presented at the Naxos school.

The basic idea of the theory of cosmological perturbations is simple. In order to obtain the action for linearized cosmological perturbations, we expand the action for gravity and matter to quadratic order in the fluctuating degrees of freedom.

The linear terms cancel because the background is taken to satisfy the background equations of motion.

At first sight, it appears that there are ten degrees of freedom for the metric fluctuations, in addition to the matter perturbations. However, four of these degrees of freedom are equivalent to space-time diffeomorphisms. To study the remaining six degrees of freedom for metric fluctuations it proves very useful to classify them according to how they transform under spatial rotations. There are two scalar modes, two vector modes and two tensor modes. At linear order in cosmological perturbation theory, scalar, vector and tensor modes decouple. For simple forms of matter such as scalar fields or perfect fluids, the matter fluctuations couple only to the scalar metric modes. These are the so-called “cosmological perturbations” which we study below.

If matter has no anisotropic stress, then one of the scalar metric degrees of freedom disappears. In addition, one of the Einstein constraint equations couples the remaining metric degree of freedom to matter. Thus, if there is only one matter component (e.g. one scalar matter field), there is only one independent scalar cosmological fluctuation mode.

To obtain the action and equation of motion for this mode, we begin with the Einstein-Hilbert action for gravity and the action for matter (which we take for simplicity to be a scalar field φ - for the more complicated case of general hydrodynamical fluctuations the reader is referred to [86])

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{16\pi G} R + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) \right], \quad (13)$$

where R is the Ricci curvature scalar.

The simplest way to proceed is to work in longitudinal gauge, in which the metric and matter take the form (assuming no anisotropic stress)

$$\begin{aligned} ds^2 &= a^2(\eta) [(1 + 2\phi(\eta, \mathbf{x})) d\eta^2 - (1 - 2\phi(\eta, \mathbf{x})) d\mathbf{x}^2] \\ \varphi(\eta, \mathbf{x}) &= \varphi_0(\eta) + \delta\varphi(\eta, \mathbf{x}), \end{aligned} \quad (14)$$

where η in conformal time. The two fluctuation variables ϕ and $\delta\varphi$ must be linked by the Einstein constraint equations since there cannot be matter fluctuations without induced metric fluctuations.

The two nontrivial tasks of the lengthy [86] computation of the quadratic piece of the action is to find out what combination of $\delta\varphi$ and ϕ gives the variable v in terms of which the action has canonical kinetic term, and what the form of the time-dependent mass is. This calculation involves inserting the ansatz (14) into the action (13), expanding the result to second order in the fluctuating fields, making use of the background and of the constraint equations, and dropping total derivative terms from the action. In the context of scalar field matter, the quantum theory of cosmological fluctuations was developed by Mukhanov [88, 89] and Sasaki [90]. The result is the following contribution $S^{(2)}$ to the action quadratic in the perturbations:

$$S^{(2)} = \frac{1}{2} \int d^4x [v'^2 - v_{,i}v_{,i} + \frac{z''}{z} v^2], \quad (15)$$

where the canonical variable v (the ‘‘Sasaki-Mukhanov variable’’ introduced in [89] - see also [91]) is given by

$$v = a \left[\delta\phi + \frac{\phi_0'}{\mathcal{H}} \phi \right], \quad (16)$$

with $\mathcal{H} = a'/a$, and where

$$z = \frac{a\phi_0'}{\mathcal{H}}. \quad (17)$$

As long as the equation of state does not change over time

$$z(\eta) \sim a(\eta). \quad (18)$$

Note that the variable v is related to the curvature perturbation \mathcal{R} in comoving coordinates introduced in [94] and closely related to the variable ζ used in [92, 93]:

$$v = z\mathcal{R}. \quad (19)$$

The equation of motion which follows from the action (15) is (in momentum space)

$$v_k'' + k^2 v_k - \frac{z''}{z} v_k = 0, \quad (20)$$

where v_k is the k 'th Fourier mode of v . The mass term in the above equation is in general given by the Hubble scale (the scale whose wave-number will be denoted k_H). Thus, it immediately follows that on small length scales, i.e. for $k > k_H$, the solutions for v_k are constant amplitude oscillations. These oscillations freeze out at Hubble radius crossing, i.e. when $k = k_H$. On longer scales ($k \ll k_H$), there is a mode of v_k which scales as z . This mode is the dominant one in an expanding universe, but not in a contracting one.

Given the action (15), the cosmological perturbations can be quantized by canonical quantization (in the same way that a scalar matter field on a fixed cosmological background is quantized [95]).

The final step in the quantum theory of cosmological perturbations is to specify an initial state. Since in inflationary cosmology all pre-existing classical fluctuations are red-shifted by the accelerated expansion of space, one usually assumes that the field v starts out at the initial time t_i mode by mode in its vacuum state. This prescription, however, can be criticized in light of the trans-Planckian problem for cosmological fluctuations. It assumes that ultraviolet modes which are continuously crossing the Planck scale cutoff $k = m_{pl}$ are in their vacuum state, which is a strong constraint on physics on trans-Planckian scales.

There are two other questions which immediately emerge: what is the initial time t_i , and which of the many possible vacuum states should be chosen. It is usually assumed that since the fluctuations only oscillate on sub-Hubble scales, the choice

of the initial time is not important, as long as it is earlier than the time when scales of cosmological interest today cross the Hubble radius during the inflationary phase. The state is usually taken to be the Bunch-Davies vacuum (see e.g. [95]), since this state is empty of particles at t_i in the coordinate frame determined by the FRW coordinates. Thus, we choose the initial conditions

$$\begin{aligned} v_k(\eta_i) &= \frac{1}{\sqrt{2k}} \\ v'_k(\eta_i) &= \frac{\sqrt{k}}{\sqrt{2}} \end{aligned} \quad (21)$$

where η_i is the conformal time corresponding to the physical time t_i .

Returning to the case of an expanding universe, the scaling

$$v_k \sim z \sim a \quad (22)$$

implies that the wave function of the quantum variable v_k which performs quantum vacuum fluctuations on sub-Hubble scales, stops oscillating on super-Hubble scales and instead is squeezed (the amplitude increases in configuration space but decreases in momentum space). This squeezing corresponds to quantum particle production. It is also one of the two conditions which are required for the classicalization of the fluctuations. The second condition is decoherence which is induced by the non-linearities in the dynamical system which are inevitable since the Einstein action leads to highly nonlinear equations (see [96] for an in-depth discussion of this point, and [97] for related work).

Note that the squeezing of cosmological fluctuations on super-Hubble scales occurs in all models, in particular in string gas cosmology and in the bouncing universe scenario since also in these scenarios perturbations propagate on super-Hubble scales for a long period of time. In a contracting phase, the dominant mode of v_k on super-Hubble scales is not the one given in (22) (which in this case is a decaying mode), but rather the second mode which scales as z^{-p} with an exponent p which is positive and whose exact value depends on the background equation of state.

Applications of this theory in inflationary cosmology, in the matter bounce scenario and in string gas cosmology will be considered in the following sections.

5 Fluctuations in Inflationary Cosmology

We will now use the quantum theory of cosmological perturbations developed in the previous section to calculate the spectrum of curvature fluctuations in inflationary cosmology. The starting point are quantum vacuum initial conditions for the canonical fluctuation variable v_k :

$$v_k(\eta_i) = \frac{1}{\sqrt{2k}} \quad (23)$$

for all k for which the wavelength is smaller than the Hubble radius at the initial time t_i .

The amplitude remains unchanged until the modes exit the Hubble radius at the respective times $t_H(k)$ given by

$$a^{-1}(t_H(k))k = H. \quad (24)$$

We need to compute the power spectrum $\mathcal{P}_{\mathcal{R}}(k)$ of the curvature fluctuation \mathcal{R} defined in (19) at some late time t when the modes are super-Hubble. We first relate the power spectrum via the growth rate (22) of ν on super-Hubble scales to the power spectrum at the time $t_H(k)$ and then use the constancy of the amplitude of ν on sub-Hubble scales to relate it to the initial conditions (23). Thus

$$\begin{aligned} \mathcal{P}_{\mathcal{R}}(k, t) &\equiv k^3 \mathcal{R}_k^2(t) = k^3 z^{-2}(t) |\nu_k(t)|^2 \\ &= k^3 z^{-2}(t) \left(\frac{a(t)}{a(t_H(k))} \right)^2 |\nu_k(t_H(k))|^2 \\ &= k^3 z^{-2}(t_H(k)) |\nu_k(t_H(k))|^2 \\ &\sim k^3 \left(\frac{a(t)}{z(t)} \right)^2 a^{-2}(t_H(k)) |\nu_k(t_i)|^2, \end{aligned} \quad (25)$$

where in the final step we have used (18) and the constancy of the amplitude of ν on sub-Hubble scales. Making use of the condition (24) for Hubble radius crossing, and of the initial conditions (23), we immediately see that

$$\mathcal{P}_{\mathcal{R}}(k, t) \sim \left(\frac{a(t)}{z(t)} \right)^2 k^3 k^{-2} k^{-1} H^2, \quad (26)$$

and that thus a scale invariant power spectrum with amplitude proportional to H^2 results, in agreement with what was argued on heuristic grounds in the overview of inflation in the first section. To obtain the precise amplitude, we need to make use of the relation between z and a . We obtain

$$\mathcal{P}_{\mathcal{R}}(k, t) \sim \frac{H^4}{\Phi_0^2} \quad (27)$$

which for any given value of k is to be evaluated at the time $t_H(k)$ (before the end of inflation). For a scalar field potential (see following subsection)

$$V(\varphi) = \lambda \varphi^4 \quad (28)$$

the resulting amplitude in (27) is λ . Thus, in order to obtain the observed value of the power spectrum of the order of 10^{-10} , the coupling constant λ must be tuned to a very small value.

6 Matter Bounce and Structure Formation

As we already discussed in Section 2 of these notes, in a non-singular bouncing cosmology fluctuations on scales relevant to current cosmological observations have a physical wavelength which at early times during the contracting phase was smaller than the Hubble radius. Hence, a causal generation mechanism for fluctuations is possible. In fact, in [17, 18] it was realized that fluctuations which originate on sub-Hubble scales in their quantum vacuum state and exit the Hubble radius during a matter-dominated contracting phase acquire a scale-invariant spectrum. As we review below, this is due to the particular growth rate of the dominant fluctuation mode in the contracting phase which is exactly right to convert a vacuum spectrum into a scale-invariant one. During any non-matter phase of contraction which might follow the initial matter-dominated phase the slope of the spectrum remains unchanged on super-Hubble scales since all corresponding mode functions grow by the same factor. Thus, the spectrum of fluctuations right before the bounce is scale-invariant. Provided that the spectrum does not change its slope during the bounce phase, a model falling into the matter bounce category will provide an alternative to inflation for generating a scale-invariant spectrum of curvature perturbations.

The propagation of infrared (IR) fluctuations through the non-singular bounce was analyzed in the case of the higher derivative gravity model of [47] in [98], in mirage cosmology in [49], in the case of the quintom bounce in [34, 37], for a ghost condensate bounce in [40], for a Horava-Lifshitz bounce in [99], and more recently [31] in the string theory bounce model of [30]. The result of these studies is that the scale-invariance of the spectrum before the bounce persists after the bounce as long as the time period of the bounce phase is short compared to the wavelengths of the modes being considered. Note that if the fluctuations have a thermal origin, then the condition on the background cosmology to yield scale-invariance of the spectrum of fluctuations is different [100].

6.1 Basics

First we will consider fluctuations in a matter bounce without extra degrees of freedom. In this case, we need only focus on the usual fluctuation variable v . The equation of motion for its Fourier mode v_k is

$$v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0. \quad (29)$$

If the equation of state of the background is time-independent, then $z \sim a$ and hence the negative square mass term in (29) is H^2 . Thus, on length scales smaller than the Hubble radius, the solutions of (29) are oscillating, whereas on larger scales they are frozen in, and their amplitude depends on the time evolution of z .

In the case of an expanding universe the dominant mode of v scales as z . However, in a contracting universe it is the second of the two modes which dominates. If the contracting phase is matter-dominated, i.e. $a(t) \sim t^{2/3}$ and $\eta(t) \sim t^{1/3}$ the dominant mode of v scales as η^{-1} and hence

$$v_k(\eta) = c_1 \eta^2 + c_2 \eta^{-1}, \quad (30)$$

where c_1 and c_2 are constants. The c_1 mode is the mode for which ζ is constant on super-Hubble scales. However, in a contracting universe it is the c_2 mode which dominates and leads to a scale-invariant spectrum [17, 18]:

$$\begin{aligned} P_\zeta(k, \eta) &\sim k^3 |v_k(\eta)|^2 a^{-2}(\eta) \\ &\sim k^3 |v_k(\eta_H(k))|^2 \left(\frac{\eta_H(k)}{\eta} \right)^2 \sim k^{3-1-2} \\ &\sim \text{const}, \end{aligned} \quad (31)$$

where we have made use of the scaling of the dominant mode of v_k , the Hubble radius crossing condition $\eta_H(k) \sim k^{-1}$, and the assumption that we have a vacuum spectrum at Hubble radius crossing.

At this point we have shown that the spectrum of fluctuations is scale-invariant on super-Hubble scales before the bounce phase. The evolution during the bounce depends in principle on the specific realization of the non-singular bounce. In any concrete model, the equations of motion can be solved numerically without approximation during the bounce. Alternatively, we can solve them approximately using analytical techniques. Key to the analytical analysis are the General Relativistic matching conditions for fluctuations across a phase transition in the background [101, 102]. These conditions imply that both Φ and $\tilde{\zeta}$ are conserved at the bounce, where

$$\tilde{\zeta} = \zeta + \frac{1}{3} \frac{k^2 \Phi}{\mathcal{H}^2 - \mathcal{H}'}. \quad (32)$$

However, as stressed in [103], these matching conditions can only be used at a transition when the background metric obeys the matching conditions. This is not the case if we were to match directly between the contracting matter phase and the expanding matter phase, as was done in early studies [104, 105, 106] of fluctuations in the Ekpyrotic scenario.

In the case of a non-singular bounce we have three phases: the initial contracting phase with a fixed equation of state (e.g. $w = 0$), a bounce phase during which the universe smoothly transits between contraction and expansion, and finally the expanding phase with constant w . We need to apply the matching conditions twice: first at the transition between the contracting phase and the bounce phase (on both sides of the matching surface the universe is contracting), and then between the bouncing phase and the expanding phase. The bottom line of the studies of [98, 49, 34, 37, 40, 99, 31] is that on length scales large compared to the time of the bounce, the spectrum of curvature fluctuations is not changed during the bounce phase. Since typically the bounce time is set by a microphysical scale whereas the

wavelength of fluctuations which we observe today is macroscopic (about 1mm if the bounce scale is set by the particle physics GUT scale), we conclude that for scales relevant to current observations the spectrum is unchanged during the bounce. This completes the demonstration that a non-singular matter bounce leads to a scale-invariant spectrum of cosmological perturbations after the bounce provided that the initial spectrum on sub-Hubble scales is vacuum.

The fact that fluctuations grow both in the contracting and expanding phase has implications for cyclic cosmologies in four space-time dimensions: In the presence of fluctuations, no such cyclic models are possible - the growth of fluctuations breaks this cyclicity. As we showed above, the spectral index of the power spectrum of the fluctuations changes during the bounce. Hence, four space-time-dimensional cyclic background cosmologies are not predictive - the index of the power spectrum changes from cycle to cycle [107]. Note that the cyclic version of the Ekpyrotic scenario [108] avoids these problems because it is not cyclic in the above sense: it is a higher space-time-dimensional model in which the radius of an extra dimensions evolves cyclically, but the four-dimensional scale factor does not.

The above analysis is applicable only as long as no new degrees of freedom become relevant at high energy densities, in particular during the bounce phase. In non-singular bounce models obtained by modifying the matter sector, new degrees of freedom arise from the extra matter fields. They can thus give entropy fluctuations which may compete with the adiabatic mode studied above. In the quintom bounce model this issue has recently been studied in [109]. It was found that fluctuations in the ghost field which yields the bounce are unimportant on large scales since they have a blue spectrum. However, entropy fluctuations due to extra low-mass fields can be important. Their spectrum is also scale-invariant, and this yields the “matter bounce curvaton” mechanism.

In non-singular bouncing models obtained by modifying the gravitational sector of the theory the identification of potential extra degrees of freedom is more difficult. As an example, let us mention the situation in the case of the Horava-Lifshitz bounce. The theory has the same number of geometric degrees of freedom as General Relativity, but less symmetries. Thus, more of the degrees of freedom are physical. Recall from the discussion of the theory of cosmological perturbations in Section 2 that there are ten total geometrical degrees of freedom for linear cosmological perturbations, four of them being scalar, four vector and two tensor. In Einstein gravity the symmetry group of space-time diffeomorphisms is generated at the level of linear fluctuations by four functions, leaving six of the ten geometrical variables as physical - two scalar, two vector and two tensor modes. In the absence of anisotropic stress the number of scalar variables is reduced by one, and the Hamiltonian constraint relates the remaining scalar metric fluctuation to matter.

In Horava-Lifshitz gravity one loses one scalar gauge degree of freedom, namely that of space-dependent time reparametrizations. Thus, one expects an extra physical degree of freedom. It has been recently shown [110] that in the projectable version of the theory (in which the lapse function $N(t)$ is constrained to be a function of time only) the extra degree of scalar cosmological perturbations is either ghost-like or tachyonic, depending on parameters in the Lagrangian. Thus, the the-

ory appears to be ill-behaved in the context of cosmology. However, in the full non-projectable version (in which the lapse $N(t, \mathbf{x})$ is a function of both space and time, the extra degree of freedom is well behaved. It is important on ultraviolet scales but decouples in the infrared [111].

6.2 Specific Predictions

Canonical single field inflation models predict very small non-Gaussianities in the spectrum of fluctuations. One way to characterize the non-Gaussianities is via the three point function of the curvature fluctuation, also called the “bispectrum”. As realized in [112], the bispectrum induced in the minimal matter bounce scenario (no entropy modes considered) has an amplitude which is at the borderline of what the Planck satellite experiment will be able to detect, and it has a special form. These are specific predictions of the matter bounce scenario with which the matter bounce scenario can be distinguished from those of standard inflationary models (see [113] for a recent detailed review of non-Gaussianities in inflationary cosmology and a list of references). In the following we give a very brief summary of the analysis of non-Gaussianities in the matter bounce scenario.

Non-Gaussianities are induced in any cosmological model simply because the Einstein equations are non-linear. In momentum space, the bispectrum contains amplitude and shape information. The bispectrum is a function of the three momenta. Momentum conservation implies that the three momenta have to add up to zero. However, this still leaves a rich shape information in the bispectrum in addition to the information about the overall amplitude.

A formalism to compute the non-Gaussianities for the curvature fluctuation variable ζ was developed in [114]. Working in the interaction representation, the three-point function of ζ is given to leading order by

$$\begin{aligned} & \langle \zeta(t, \mathbf{k}_1) \zeta(t, \mathbf{k}_2) \zeta(t, \mathbf{k}_3) \rangle \\ &= i \int_{t_i}^t dt' \langle [\zeta(t, \mathbf{k}_1) \zeta(t, \mathbf{k}_2) \zeta(t, \mathbf{k}_3), L_{int}(t')] \rangle, \end{aligned} \quad (33)$$

where t_i corresponds to the initial time before which there are any non-Gaussianities. The square parentheses indicate the commutator, and L_{int} is the interaction Lagrangian

The interaction Lagrangian contains many terms. In particular, there are terms containing the time derivative of ζ . Each term leads to a particular shape of the bispectrum. In an expanding universe such as in inflationary cosmology $\dot{\zeta} = 0$. However, in a contracting phase the time derivative of ζ does not vanish since the dominant mode is growing in time. Hence, there are new contributions to the shape which have a very different form from the shape of the terms which appear in inflationary cosmology. The larger value of the amplitude of the bispectrum follows again from the fact that there is a mode function which grows in time in the contracting phase.

The three-point function can be expressed in the following general form:

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3) \rangle = (2\pi)^7 \delta(\sum \mathbf{k}_i) \frac{P_\zeta^2}{\prod k_i^3} \times \mathcal{A}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3), \quad (34)$$

where $k_i = |\mathbf{k}_i|$ and \mathcal{A} is the shape function. In this expression we have factored out the dependence on the power spectrum P_ζ . In inflationary cosmology it has become usual to express the bispectrum in terms of a non-Gaussianity parameter f_{NL} . However, this is only useful if the shape of the three point function is known. As a generalization, we here use [112]

$$|\mathcal{B}|_{NL}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{10}{3} \frac{\mathcal{A}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)}{\sum_i k_i^3}. \quad (35)$$

The computation of the bispectrum is tedious. In the case of the matter bounce (no entropy fluctuations) the result is

$$\begin{aligned} \mathcal{A} = \frac{3}{256 \prod k_i^2} & \left\{ 3 \sum k_i^9 + \sum_{i \neq j} k_i^7 k_j^2 \right. \\ & - 9 \sum_{i \neq j} k_i^6 k_j^3 + 5 \sum_{i \neq j} k_i^5 k_j^4 \\ & \left. - 66 \sum_{i \neq j \neq k} k_i^5 k_j^2 k_k^2 + 9 \sum_{i \neq j \neq k} k_i^4 k_j^3 k_k^2 \right\}. \end{aligned} \quad (36)$$

This equation describes the shape which is predicted. Some of the terms (such as the last two) are the same as those which occur in single field slow-roll inflation, but the others are new. Note, in particular, that the new terms are not negligible.

If we project the resulting shape function \mathcal{A} onto some popular shape masks we

$$|\mathcal{B}|_{NL}^{\text{local}} = -\frac{35}{8}, \quad (37)$$

for the local shape ($k_1 \ll k_2 = k_3$). This is negative and of order $O(1)$. For the equilateral form ($k_1 = k_2 = k_3$) the result is

$$|\mathcal{B}|_{NL}^{\text{equil}} = -\frac{255}{64}, \quad (38)$$

and for the folded form ($k_1 = 2k_2 = 2k_3$) one obtains the value

$$|\mathcal{B}|_{NL}^{\text{folded}} = -\frac{9}{4}. \quad (39)$$

These amplitudes are close to what the Planck CMB satellite experiment will be able to detect.

The matter bounce scenario also predicts a change in the slope of the primordial power spectrum on small scales [115]: scales which exit the Hubble radius in the radiation phase retain a blue spectrum since the squeezing rate on scales larger than the Hubble radius is insufficient to give longer wavelength modes a sufficient boost relative to the shorter wavelength ones.

7 String Gas Cosmology and Structure Formation

In this section we discuss cosmological fluctuations in one particular realization of the emergent universe scenario, namely string gas cosmology. In contrast to the case of exponential inflation and the matter bounce, where a scale-invariant spectrum emerges from initial quantum vacuum fluctuations independent of the specific realization of the background cosmology, in the case of the emergent universe scenario a scale-invariant spectrum is generated only in the string gas cosmology realization, and in other realizations which share some general properties which will be mentioned at the end of this section.

7.1 Overview

The analysis of cosmological perturbations in string gas cosmology (pioneered in [14]) is based on the cosmological background of string gas cosmology represented in Figure 6. In turn, this background yields the space-time diagram sketched in Figure 10. As in Figure 2, the vertical axis is time and the horizontal axis denotes the physical distance. For times $t < t_R$, we are in the static Hagedorn phase and the Hubble radius is infinite. For $t > t_R$, the Einstein frame Hubble radius is expanding as in standard cosmology. The time t_R is when the string winding modes begin to decay into string loops, and the scale factor starts to increase, leading to the transition to the radiation phase of standard cosmology.

Let us now compare the evolution of the physical wavelength corresponding to a fixed co-moving scale with that of the Einstein frame Hubble radius $H^{-1}(t)$. The evolution of scales in string gas cosmology is identical to the evolution in standard and in inflationary cosmology for $t > t_R$. If we follow the physical wavelength of the co-moving scale which corresponds to the current Hubble radius back to the time t_R , then - taking the Hagedorn temperature to be of the order 10^{16} GeV - we obtain a length of about 1 mm. Compared to the string scale and the Planck scale, this is in the far infrared.

The physical wavelength is constant in the Hagedorn phase since space is static. But, as we enter the Hagedorn phase going back in time, the Hubble radius diverges to infinity. Hence, as in the case of inflationary cosmology, fluctuation modes begin sub-Hubble during the Hagedorn phase, and thus a causal generation mechanism for fluctuations is possible.

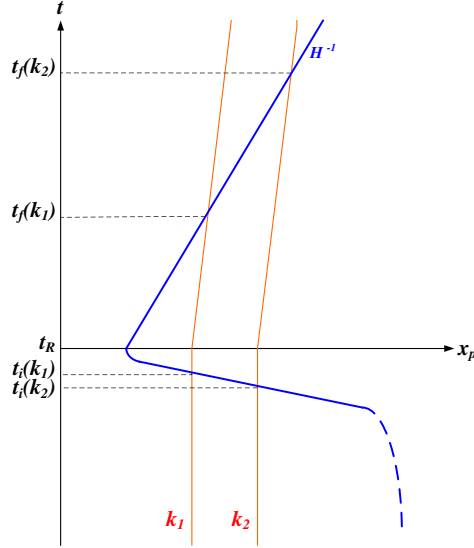


Fig. 10 Space-time diagram (sketch) showing the evolution of fixed co-moving scales in string gas cosmology. The vertical axis is time, the horizontal axis is physical distance. The solid curve represents the Einstein frame Hubble radius H^{-1} which shrinks abruptly to a micro-physical scale at t_R and then increases linearly in time for $t > t_R$. Fixed co-moving scales (the dotted lines labeled by k_1 and k_2) which are currently probed in cosmological observations have wavelengths which are smaller than the Hubble radius long before t_R . They exit the Hubble radius at times $t_i(k)$ just prior to t_R , and propagate with a wavelength larger than the Hubble radius until they re-enter the Hubble radius at times $t_f(k)$.

However, the physics of the generation mechanism is very different. In the case of inflationary cosmology, fluctuations are assumed to start as quantum vacuum perturbations because classical inhomogeneities are red-shifting. In contrast, in the Hagedorn phase of string gas cosmology there is no red-shifting of classical matter. Hence, it is the fluctuations in the classical matter which dominate. Since classical matter is a string gas, the dominant fluctuations are string thermodynamic fluctuations.

Our proposal for string gas structure formation is the following [14] (see [116] for a more detailed description). For a fixed co-moving scale with wavenumber k we compute the matter fluctuations while the scale is sub-Hubble (and therefore gravitational effects are sub-dominant). When the scale exits the Hubble radius at time $t_i(k)$ we use the gravitational constraint equations to determine the induced metric fluctuations, which are then propagated to late times using the usual equations of

gravitational perturbation theory. Since the scales we are interested in are in the far infrared, we use the Einstein constraint equations for fluctuations.

7.2 Spectrum of Cosmological Fluctuations

We write the metric including cosmological perturbations (scalar metric fluctuations) and gravitational waves in the following form (using conformal time η)

$$ds^2 = a^2(\eta) \{ (1 + 2\Phi) d\eta^2 - [(1 - 2\Phi)\delta_{ij} + h_{ij}] dx^i dx^j \} . \quad (40)$$

As in previous sections, we are working in the longitudinal gauge for the scalar metric perturbations and we have taken matter to be free of anisotropic stress. The spatial tensor $h_{ij}(\mathbf{x}, t)$ is transverse and traceless and represents the gravitational waves.

Note that in contrast to the case of slow-roll inflation, scalar metric fluctuations and gravitational waves are generated by matter at the same order in cosmological perturbation theory. This could lead to the expectation that the amplitude of gravitational waves in string gas cosmology should be generically larger than in inflationary cosmology. This expectation, however, is not realized [117] since there is a different mechanism which suppresses the gravitational waves relative to the density perturbations (namely the fact that the gravitational wave amplitude is set by the amplitude of the pressure, and the pressure is suppressed relative to the energy density in the Hagedorn phase).

Assuming that the fluctuations are described by the perturbed Einstein equations (they are *not* if the dilaton is not fixed [118, 119]), then the spectra of cosmological perturbations Φ and gravitational waves h are given by the energy-momentum fluctuations in the following way [116]

$$\langle |\Phi(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0_0(k) \delta T^0_0(k) \rangle , \quad (41)$$

where the pointed brackets indicate expectation values, and

$$\langle |h(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i_j(k) \delta T^i_j(k) \rangle , \quad (42)$$

where on the right hand side of (42) we mean the average over the correlation functions with $i \neq j$, and h is the amplitude of the gravitational waves ².

Let us now use (41) to determine the spectrum of scalar metric fluctuations. We first calculate the root mean square energy density fluctuations in a sphere of radius $R = k^{-1}$. For a system in thermal equilibrium they are given by the specific heat capacity C_V via

$$\langle \delta \rho^2 \rangle = \frac{T^2}{R^6} C_V . \quad (43)$$

² The gravitational wave tensor h_{ij} can be written as the amplitude h multiplied by a constant polarization tensor.

The specific heat of a gas of closed strings on a torus of radius R can be derived from the partition function of a gas of closed strings. This computation was carried out in [120] (see also [121]) with the result

$$C_V \approx 2 \frac{R^2/\ell^3}{T(1-T/T_H)}. \quad (44)$$

The specific heat capacity scales holographically with the size of the box. This result follows rigorously from evaluating the string partition function in the Hagedorn phase. The result, however, can also be understood heuristically: in the Hagedorn phase the string winding modes are crucial. These modes look like point particles in one less spatial dimension. Hence, we expect the specific heat capacity to scale like in the case of point particles in one less dimension of space³.

With these results, the power spectrum $P(k)$ of scalar metric fluctuations can be evaluated as follows

$$\begin{aligned} P_\Phi(k) &\equiv \frac{1}{2\pi^2} k^3 |\Phi(k)|^2 \\ &= 8G^2 k^{-1} \langle |\delta\rho(k)|^2 \rangle \\ &= 8G^2 k^2 \langle (\delta M)^2 \rangle_R \\ &= 8G^2 k^{-4} \langle (\delta\rho)^2 \rangle_R \\ &= 8G^2 \frac{T}{\ell_s^3} \frac{1}{1-T/T_H}, \end{aligned} \quad (45)$$

where in the first step we have used (41) to replace the expectation value of $|\Phi(k)|^2$ in terms of the correlation function of the energy density, and in the second step we have made the transition to position space.

The first conclusion from the result (45) is that the spectrum is approximately scale-invariant ($P(k)$ is independent of k). It is the ‘holographic’ scaling $C_V(R) \sim R^2$ which is responsible for the overall scale-invariance of the spectrum of cosmological perturbations. However, there is a small k dependence which comes from the fact that in the above equation for a scale k the temperature T is to be evaluated at the time $t_i(k)$. Thus, the factor $(1 - T/T_H)$ in the denominator is responsible for giving the spectrum a slight dependence on k . Since the temperature slightly decreases as time increases at the end of the Hagedorn phase, shorter wavelengths for which $t_i(k)$ occurs later obtain a smaller amplitude. Thus, the spectrum has a slight red tilt.

³ We emphasize that it was important for us to have compact spatial dimensions in order to obtain the winding modes which are crucial to get the holographic scaling of the thermodynamic quantities.

7.3 Key Prediction of String Gas Cosmology

As discovered in [117], the spectrum of gravitational waves is also nearly scale invariant. However, in the expression for the spectrum of gravitational waves the factor $(1 - T/T_H)$ appears in the numerator, thus leading to a slight blue tilt in the spectrum. This is a prediction with which the cosmological effects of string gas cosmology can be distinguished from those of inflationary cosmology, where quite generically a slight red tilt for gravitational waves results. The physical reason for the blue tilt in string gas cosmology is that large scales exit the Hubble radius earlier when the pressure and hence also the off-diagonal spatial components of $T_{\mu\nu}$ are closer to zero.

Let us analyze this issue in a bit more detail and estimate the dimensionless power spectrum of gravitational waves. First, we make some general comments. In slow-roll inflation, to leading order in perturbation theory matter fluctuations do not couple to tensor modes. This is due to the fact that the matter background field is slowly evolving in time and the leading order gravitational fluctuations are linear in the matter fluctuations. In our case, the background is not evolving (at least at the level of our computations), and hence the dominant metric fluctuations are quadratic in the matter field fluctuations. At this level, matter fluctuations induce both scalar and tensor metric fluctuations. Based on this consideration we might expect that in our string gas cosmology scenario, the ratio of tensor to scalar metric fluctuations will be larger than in simple slow-roll inflationary models. However, since the amplitude h of the gravitational waves is proportional to the pressure, and the pressure is suppressed in the Hagedorn phase, the amplitude of the gravitational waves will also be small in string gas cosmology.

The method for calculating the spectrum of gravitational waves is similar to the procedure outlined in the last section for scalar metric fluctuations. For a mode with fixed co-moving wavenumber k , we compute the correlation function of the off-diagonal spatial elements of the string gas energy-momentum tensor at the time $t_i(k)$ when the mode exits the Hubble radius and use (42) to infer the amplitude of the power spectrum of gravitational waves at that time. We then follow the evolution of the gravitational wave power spectrum on super-Hubble scales for $t > t_i(k)$ using the equations of general relativistic perturbation theory.

The power spectrum of the tensor modes is given by (42):

$$P_h(k) = 16\pi^2 G^2 k^{-4} k^3 \langle \delta T^i_j(k) \delta T^i_j(k) \rangle \quad (46)$$

for $i \neq j$. Note that the k^3 factor multiplying the momentum space correlation function of T^i_j gives the position space correlation function $C^i_j{}^i_j(R)$, namely the root mean square fluctuation of T^i_j in a region of radius $R = k^{-1}$. Thus,

$$P_h(k) = 16\pi^2 G^2 k^{-4} C^i_j{}^i_j(R). \quad (47)$$

The correlation function $C^i_j{}^i_j$ on the right hand side of the above equation follows from the thermal closed string partition function and was computed explicitly in

[121] (see also [116]). We obtain

$$P_h(k) \sim 16\pi^2 G^2 \frac{T}{l_s^3} (1 - T/T_H) \ln^2 \left[\frac{1}{l_s^2 k^2} (1 - T/T_H) \right], \quad (48)$$

which, for temperatures close to the Hagedorn value reduces to

$$P_h(k) \sim \left(\frac{l_{Pl}}{l_s} \right)^4 (1 - T/T_H) \ln^2 \left[\frac{1}{l_s^2 k^2} (1 - T/T_H) \right]. \quad (49)$$

This shows that the spectrum of tensor modes is - to a first approximation, namely neglecting the logarithmic factor and neglecting the k -dependence of $T(t_i(k))$ - scale-invariant.

On super-Hubble scales, the amplitude h of the gravitational waves is constant. The wave oscillations freeze out when the wavelength of the mode crosses the Hubble radius. As in the case of scalar metric fluctuations, the waves are squeezed. Whereas the wave amplitude remains constant, its time derivative decreases. Another way to see this squeezing is to change variables to

$$\psi(\eta, \mathbf{x}) = a(\eta)h(\eta, \mathbf{x}) \quad (50)$$

in terms of which the action has canonical kinetic term. The action in terms of ψ becomes

$$S = \frac{1}{2} \int d^4x \left(\psi'^2 - \psi_{,i}\psi_{,i} + \frac{a''}{a} \psi^2 \right) \quad (51)$$

from which it immediately follows that on super-Hubble scales $\psi \sim a$. This is the squeezing of gravitational waves [122].

Since there is no k -dependence in the squeezing factor, the scale-invariance of the spectrum at the end of the Hagedorn phase will lead to a scale-invariance of the spectrum at late times.

Note that in the case of string gas cosmology, the squeezing factor $z(\eta)$ for scalar metric fluctuations does not differ substantially from the squeezing factor $a(\eta)$ for gravitational waves. In the case of inflationary cosmology, $z(\eta)$ and $a(\eta)$ differ greatly during reheating, leading to a much larger squeezing for scalar metric fluctuations, and hence to a suppressed tensor to scalar ratio of fluctuations. In the case of string gas cosmology, there is no difference in squeezing between the scalar and the tensor modes.

Let us return to the discussion of the spectrum of gravitational waves. The result for the power spectrum is given in (49), and we mentioned that to a first approximation this corresponds to a scale-invariant spectrum. As realized in [117], the logarithmic term and the k -dependence of $T(t_i(k))$ both lead to a small blue-tilt of the spectrum. This feature is characteristic of our scenario and cannot be reproduced in inflationary models. In inflationary models, the amplitude of the gravitational waves is set by the Hubble constant H . The Hubble constant cannot increase during inflation, and hence no blue tilt of the gravitational wave spectrum is possible.

A heuristic way of understanding the origin of the slight blue tilt in the spectrum of tensor modes is as follows. The closer we get to the Hagedorn temperature, the more the thermal bath is dominated by long string states, and thus the smaller the pressure will be compared to the pressure of a pure radiation bath. Since the pressure terms (strictly speaking the anisotropic pressure terms) in the energy-momentum tensor are responsible for the tensor modes, we conclude that the smaller the value of the wavenumber k (and thus the higher the temperature $T(t_i(k))$ when the mode exits the Hubble radius, the lower the amplitude of the tensor modes. In contrast, the scalar modes are determined by the energy density, which increases at $T(t_i(k))$ as k decreases, leading to a slight red tilt.

The reader may ask about the predictions of string gas cosmology for non-Gaussianities. The answer is [123] that the non-Gaussianities from the thermal string gas perturbations are Poisson-suppressed on scales larger than the thermal wavelength in the Hagedorn phase. However, if the spatial sections are initially large, then it is possible that a network of cosmic superstrings [124] will be left behind. These strings - if stable - would achieve a scaling solution (constant number of strings crossing each Hubble volume at each time [69, 70, 71]). Such strings give rise to linear discontinuities in the CMB temperature maps [125], lines which can be searched for using edge detection algorithms such as the Canny algorithm (see [126] for recent feasibility studies).

7.4 Comments

At the outset of this section we mentioned that not all emergent universe scenarios will produce a scale-invariant spectrum. For example, string gas cosmology in a non-compact three-dimensional space will not have the holographic scaling of the specific heat capacity and hence will not yield a scale-invariant spectrum.

Under which conditions does our above analysis generalize? Three conditions appear to be necessary in order to obtain scale-invariant cosmological fluctuations from an emergent background. Firstly, the background cosmology should have a quasi-static early phase followed after a short transition period by the radiation phase of Standard Big Bang cosmology. Secondly, the evolution of cosmological fluctuations on the infrared scales relevant to current cosmological observations should be describable in terms of perturbed Einstein gravity, i.e. using the formalism discussed in Section 4, even if the background cosmology cannot. Finally, the specific heat capacity $C_V(R)$ in a region of radius R should scale holographically, i.e.

$$C_V(R) \sim R^2. \quad (52)$$

8 Conclusions

In these lectures I have given an overview of the matter bounce and emergent universe scenarios of primordial cosmology. Both yield causal mechanisms for the generation of a scale-invariant spectrum of cosmological perturbations, the same kind of spectrum which is predicted by inflationary cosmology. In all three scenarios, the fluctuations are to a good approximation Gaussian. Thus, current cosmological observations cannot tell these models apart.

I have discussed specific predictions for future observations with which the three early universe can be teased apart observationally. The string gas cosmology realization of the emergent Universe predicts a small blue tilt in the spectrum of gravitational waves. Since inflationary models generically predict a red tilt, the tilt in the gravitational wave spectrum is a very promising characteristic. The simplest realization of the matter bounce scenario produces a distinguished shape of the cosmological bispectrum - and this appears to be an interesting distinctive signal to explore.

Part of the motivation for looking for alternatives to inflation comes from the realization that (at least current versions of) inflationary cosmology suffers from a number of conceptual problems, in particular a trans-Planckian problem for the fluctuations, and the singularity problem for the background cosmology. As I hope to have convinced the reader, these problems are resolved both in string gas cosmology and in the matter bounce scenario (in the latter, the singularity problem is “solved” by construction).

The matter bounce and emergent universe scenarios successfully address many of the problems of Standard Big Bang cosmology which inflationary cosmology addresses. In particular, neither scenario has a horizon problem. However, they do not solve all of the problems. The biggest challenge for the matter bounce scenario appears to be the instability to anisotropic stress. The biggest problem of string gas cosmology is our lack of an effective field theory which is consistent with the cosmological background evolution which is required. String gas cosmology also does not explain the size and entropy of the universe.

I have focused on two alternative cosmological scenarios. As mentioned earlier, there are more, e.g. the Ekpyrotic universe. A goal of future research should be to find improved realizations of all three cosmological scenarios considered here, and also to develop new paradigms which hopefully will have fewer conceptual problems than current ones.

Acknowledgements I wish to thank the organizers of this school for inviting me to give these lectures on the beautiful island of Naxos, and for their hospitality. I wish to thank all of my collaborators on whose work I have drawn. This work has been supported in part by funds from an NSERC Discovery Grant and from the Canada Research Chair program. I also acknowledge support from the Killam foundation for the period 9/09 - 8/11.

References

1. Guth AH, "The Inflationary Universe: A Possible Solution To The Horizon And Flatness Problems," *Phys. Rev. D* **23**, 347 (1981).
2. R. Brout, F. Englert and E. Gunzig, "The Creation Of The Universe As A Quantum Phenomenon," *Annals Phys.* **115**, 78 (1978).
3. A. A. Starobinsky, "A New Type Of Isotropic Cosmological Models Without Singularity," *Phys. Lett. B* **91**, 99 (1980).
4. K. Sato, "First Order Phase Transition Of A Vacuum And Expansion Of The Universe," *Mon. Not. Roy. Astron. Soc.* **195**, 467 (1981).
5. V. Mukhanov and G. Chibisov, "Quantum Fluctuation And Nonsingular Universe. (In Russian)," *JETP Lett.* **33**, 532 (1981) [*Pisma Zh. Eksp. Teor. Fiz.* **33**, 549 (1981)].
6. W. Press, "Spontaneous production of the Zel'dovich spectrum of cosmological fluctuations", *Phys. Scr.* **21**, 702 (1980).
7. A. A. Starobinsky, "Spectrum of relict gravitational radiation and the early state of the universe," *JETP Lett.* **30**, 682 (1979) [*Pisma Zh. Eksp. Teor. Fiz.* **30**, 719 (1979)].
8. Y. B. Zeldovich, "A Hypothesis, unifying the structure and the entropy of the universe," *Mon. Not. Roy. Astron. Soc.* **160**, 1P (1972).
9. E. R. Harrison, "Fluctuations at the threshold of classical cosmology," *Phys. Rev. D* **1**, 2726 (1970).
10. R. A. Sunyaev, Y. B. Zeldovich, "Small scale fluctuations of relic radiation," *Astrophys. Space Sci.* **7**, 3-19 (1970).
11. P. J. E. Peebles, J. T. Yu, "Primeval adiabatic perturbation in an expanding universe," *Astrophys. J.* **162**, 815-836 (1970).
12. C. L. Bennett *et al.*, "First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Preliminary Maps and Basic Results," *Astrophys. J. Suppl.* **148**, 1 (2003) [arXiv:astro-ph/0302207].
13. R. H. Brandenberger and C. Vafa, "Superstrings in the Early Universe," *Nucl. Phys. B* **316**, 391 (1989).
14. A. Nayeri, R. H. Brandenberger and C. Vafa, "Producing a scale-invariant spectrum of perturbations in a Hagedorn phase of string cosmology," *Phys. Rev. Lett.* **97**, 021302 (2006) [arXiv:hep-th/0511140].
15. R. H. Brandenberger, "String Gas Cosmology," publ. in "String Cosmology" (ed. by J. Erdmenger, Wiley, 2009) [arXiv:0808.0746 [hep-th]];
T. Battefeld and S. Watson, "String gas cosmology," *Rev. Mod. Phys.* **78**, 435 (2006) [arXiv:hep-th/0510022].;
R. H. Brandenberger, "String Gas Cosmology: Progress and Problems," *Class. Quant. Grav.* **28**, 204005 (2011) [arXiv:1105.3247 [hep-th]].
16. G. F. R. Ellis and R. Maartens, "The emergent universe: Inflationary cosmology with no singularity," *Class. Quant. Grav.* **21**, 223 (2004) [gr-qc/0211082].
17. D. Wands, "Duality invariance of cosmological perturbation spectra," *Phys. Rev. D* **60**, 023507 (1999) [arXiv:gr-qc/9809062].
18. F. Finelli and R. Brandenberger, "On the generation of a scale-invariant spectrum of adiabatic fluctuations in cosmological models with a contracting phase," *Phys. Rev. D* **65**, 103522 (2002) [arXiv:hep-th/0112249].
19. R. H. Brandenberger, "Cosmology of the Very Early Universe," *AIP Conf. Proc.* **1268**, 3-70 (2010). [arXiv:1003.1745 [hep-th]].
20. M. Gasperini and G. Veneziano, "Pre - big bang in string cosmology," *Astropart. Phys.* **1**, 317 (1993) [arXiv:hep-th/9211021].
21. J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, "The ekpyrotic universe: Colliding branes and the origin of the hot big bang," *Phys. Rev. D* **64**, 123522 (2001) [arXiv:hep-th/0103239].
22. V. A. Rubakov, "Harrison-Zeldovich spectrum from conformal invariance," *JCAP* **0909**, 030 (2009) [arXiv:0906.3693 [hep-th]];

- K. Hinterbichler and J. Khoury, “The Pseudo-Conformal Universe: Scale Invariance from Spontaneous Breaking of Conformal Symmetry,” arXiv:1106.1428 [hep-th].
23. F. L. Bezrukov and M. Shaposhnikov, “The Standard Model Higgs boson as the inflaton,” *Phys. Lett. B* **659**, 703 (2008) [arXiv:0710.3755 [hep-th]].
 24. F. C. Adams, K. Freese and A. H. Guth, “Constraints on the scalar field potential in inflationary models,” *Phys. Rev. D* **43**, 965 (1991).
 25. R. H. Brandenberger, “Inflationary cosmology: Progress and problems,” arXiv:hep-ph/9910410.
 26. R. H. Brandenberger and J. Martin, “The Robustness of inflation to changes in superPlanck scale physics,” *Mod. Phys. Lett. A* **16**, 999 (2001), [arXiv:astro-ph/0005432];
J. Martin and R. H. Brandenberger, “The TransPlanckian problem of inflationary cosmology,” *Phys. Rev. D* **63**, 123501 (2001), [arXiv:hep-th/0005209].
 27. J. C. Niemeyer, “Inflation with a high frequency cutoff,” *Phys. Rev. D* **63**, 123502 (2001), [arXiv:astro-ph/0005533];
J. C. Niemeyer and R. Parentani, “Minimal modifications of the primordial power spectrum from an adiabatic short distance cutoff,” *Phys. Rev. D* **64**, 101301 (2001), [arXiv:astro-ph/0101451];
S. Shankaranarayanan, “Is there an imprint of Planck scale physics on inflationary cosmology?,” *Class. Quant. Grav.* **20**, 75 (2003) [arXiv:gr-qc/0203060].
 28. S. W. Hawking and G. F. R. Ellis, “The Large scale structure of space-time,” *Cambridge University Press, Cambridge, 1973*
 29. A. Borde and A. Vilenkin, “Eternal inflation and the initial singularity,” *Phys. Rev. Lett.* **72**, 3305 (1994) [arXiv:gr-qc/9312022].
 30. C. Kounnas, H. Partouche and N. Toumbas, “Thermal duality and non-singular cosmology in d-dimensional superstrings,” *Nucl. Phys. B* **855**, 280 (2012) [arXiv:1106.0946 [hep-th]];
C. Kounnas, H. Partouche and N. Toumbas, “S-brane to thermal non-singular string cosmology,” arXiv:1111.5816 [hep-th].
 31. R. Brandenberger, C. Kounnas, H. Partouche, S. Patil and N. Toumbas, “Fluctuations in Non-Singular Bouncing Cosmologies from Type II Superstrings”, to be submitted.
 32. M. Novello and S. E. P. Bergliaffa, “Bouncing Cosmologies,” *Phys. Rept.* **463**, 127 (2008) [arXiv:0802.1634 [astro-ph]].
 33. B. Feng, X. L. Wang and X. M. Zhang, “Dark Energy Constraints from the Cosmic Age and Supernova,” *Phys. Lett. B* **607**, 35 (2005) [arXiv:astro-ph/0404224];
B. Feng, M. Li, Y. S. Piao and X. Zhang, “Oscillating quintom and the recurrent universe,” *Phys. Lett. B* **634**, 101 (2006) [arXiv:astro-ph/0407432];
Y. F. Cai, T. Qiu, Y. S. Piao, M. Li and X. Zhang, “Bouncing Universe with Quintom Matter,” *JHEP* **0710**, 071 (2007) [arXiv:0704.1090 [gr-qc]];
Y. F. Cai, T. T. Qiu, J. Q. Xia and X. Zhang, “A Model Of Inflationary Cosmology Without Singularity,” *Phys. Rev. D* **79**, 021303 (2009) [arXiv:0808.0819 [astro-ph]].
 34. Y. F. Cai, T. Qiu, R. Brandenberger, Y. S. Piao and X. Zhang, “On Perturbations of Quintom Bounce,” *JCAP* **0803**, 013 (2008) [arXiv:0711.2187 [hep-th]];
Y. F. Cai and X. Zhang, “Evolution of Metric Perturbations in Quintom Bounce model,” *JCAP* **0906**, 003 (2009) [arXiv:0808.2551 [astro-ph]].
 35. J. M. Cline, S. Jeon and G. D. Moore, “The phantom menaced: Constraints on low-energy effective ghosts,” *Phys. Rev. D* **70**, 043543 (2004) [arXiv:hep-ph/0311312].
 36. B. Grinstein, D. O’Connell and M. B. Wise, “The Lee-Wick standard model,” *Phys. Rev. D* **77**, 025012 (2008) [arXiv:0704.1845 [hep-ph]].
 37. Y. F. Cai, T. Qiu, R. Brandenberger and X. Zhang, “A Nonsingular Cosmology with a Scale-Invariant Spectrum of Cosmological Perturbations from Lee-Wick Theory,” *Phys. Rev. D* **80**, 023511 (2009) [arXiv:0810.4677 [hep-th]].
 38. J. Karouby and R. Brandenberger, “A Radiation Bounce from the Lee-Wick Construction?,” *Phys. Rev. D* **82**, 063532 (2010) [arXiv:1004.4947 [hep-th]];
J. Karouby, T. Qiu and R. Brandenberger, “On the Instability of the Lee-Wick Bounce,” *Phys. Rev. D* **84**, 043505 (2011) [arXiv:1104.3193 [hep-th]].

39. V. A. Belinsky, I. M. Khalatnikov and E. M. Lifshitz, "Oscillatory approach to a singular point in the relativistic cosmology," *Adv. Phys.* **19**, 525 (1970).
40. C. Lin, R. H. Brandenberger and L. P. Levasseur, "A Matter Bounce By Means of Ghost Condensation," *JCAP* **1104**, 019 (2011) [arXiv:1007.2654 [hep-th]].
41. P. Creminelli and L. Senatore, "A smooth bouncing cosmology with scale invariant spectrum," *JCAP* **0711**, 010 (2007) [arXiv:hep-th/0702165].
42. E. I. Buchbinder, J. Khoury and B. A. Ovrut, "New Ekpyrotic Cosmology," *Phys. Rev. D* **76**, 123503 (2007) [arXiv:hep-th/0702154].
43. T. Qiu, J. Evslin, Y. F. Cai, M. Li and X. Zhang, "Bouncing Galileon Cosmologies," *JCAP* **1110**, 036 (2011) [arXiv:1108.0593 [hep-th]];
D. A. Easson, I. Sawicki and A. Vikman, "G-Bounce," *JCAP* **1111**, 021 (2011) [arXiv:1109.1047 [hep-th]].
44. J. K. Erickson, D. H. Wesley, P. J. Steinhardt and N. Turok, "Kasner and mixmaster behavior in universes with equation of state $w = 1$," *Phys. Rev. D* **69**, 063514 (2004) [arXiv:hep-th/0312009].
45. Y. Cai, D. Easson and R. Brandenberger, "A non-singular Ekpyrotic matter bounce", to be submitted.
46. R. H. Brandenberger, V. F. Mukhanov and A. Sornborger, "A Cosmological theory without singularities," *Phys. Rev. D* **48**, 1629 (1993) [arXiv:gr-qc/9303001].
47. T. Biswas, A. Mazumdar and W. Siegel, "Bouncing universes in string-inspired gravity," *JCAP* **0603**, 009 (2006) [arXiv:hep-th/0508194].
48. A. Kehagias and E. Kiritsis, "Mirage cosmology," *JHEP* **9911**, 022 (1999) [arXiv:hep-th/9910174].
49. R. Brandenberger, H. Firouzjahi and O. Saremi, "Cosmological Perturbations on a Bouncing Brane," *JCAP* **0711**, 028 (2007) [arXiv:0707.4181 [hep-th]].
50. P. Horava, "Quantum Gravity at a Lifshitz Point," *Phys. Rev. D* **79**, 084008 (2009) [arXiv:0901.3775 [hep-th]].
51. R. Brandenberger, "Matter Bounce in Horava-Lifshitz Cosmology," *Phys. Rev. D* **80**, 043516 (2009) [arXiv:0904.2835 [hep-th]].
52. M. Bojowald, "Quantum Cosmology," *Lect. Notes Phys.* **835**, 1 (2011);
A. Ashtekar and P. Singh, "Loop Quantum Cosmology: A Status Report," *Class. Quant. Grav.* **28**, 213001 (2011) [arXiv:1108.0893 [gr-qc]].
53. P. Creminelli, A. Nicolis and E. Trincherini, "Galilean Genesis: An Alternative to inflation," *JCAP* **1011**, 021 (2010) [arXiv:1007.0027 [hep-th]].
54. L. Levasseur Perreault, R. Brandenberger and A. -C. Davis, "Defrosting in an Emergent Galileon Cosmology," *Phys. Rev. D* **84**, 103512 (2011) [arXiv:1105.5649 [astro-ph.CO]].
55. J. Kripfganz and H. Perl, "Cosmological Impact Of Winding Strings," *Class. Quant. Grav.* **5**, 453 (1988).
56. R. Hagedorn, "Statistical Thermodynamics Of Strong Interactions At High-Energies," *Nuovo Cim. Suppl.* **3**, 147 (1965).
57. J. Polchinski, *String Theory, Vols. 1 and 2*, (Cambridge Univ. Press, Cambridge, 1998).
58. T. Boehm and R. Brandenberger, "On T-duality in brane gas cosmology," *JCAP* **0306**, 008 (2003) [arXiv:hep-th/0208188].
59. K. Hotta, K. Kikkawa and H. Kunitomo, "Correlation between momentum modes and winding modes in Brandenberger-Vafa's string cosmological model," *Prog. Theor. Phys.* **98**, 687 (1997) [arXiv:hep-th/9705099].
60. M. A. R. Osorio and M. A. Vazquez-Mozo, "A Cosmological Interpretation Of Duality," *Phys. Lett. B* **320**, 259 (1994) [arXiv:hep-th/9311080].
61. A. A. Tseytlin and C. Vafa, "Elements of string cosmology," *Nucl. Phys. B* **372**, 443 (1992) [arXiv:hep-th/9109048].
62. G. Veneziano, "Scale factor duality for classical and quantum strings," *Phys. Lett. B* **265**, 287 (1991).
63. A. A. Tseytlin, "Dilaton, winding modes and cosmological solutions," *Class. Quant. Grav.* **9**, 979 (1992) [arXiv:hep-th/9112004].

64. R. H. Brandenberger, A. R. Frey and S. Kanno, "Towards A Nonsingular Tachyonic Big Crunch," *Phys. Rev. D* **76**, 063502 (2007) [arXiv:0705.3265 [hep-th]].
65. S. Arapoglu, A. Karakci and A. Kaya, "S-duality in string gas cosmology," *Phys. Lett. B* **645**, 255 (2007) [arXiv:hep-th/0611193].
66. M. Sakellariadou, "Numerical Experiments in String Cosmology," *Nucl. Phys. B* **468**, 319 (1996) [arXiv:hep-th/9511075].
67. G. B. Cleaver and P. J. Rosenthal, "String cosmology and the dimension of space-time," *Nucl. Phys. B* **457**, 621 (1995) [arXiv:hep-th/9402088].
68. R. Brandenberger, D. A. Easson and D. Kimberly, "Loitering phase in brane gas cosmology," *Nucl. Phys. B* **623**, 421 (2002) [arXiv:hep-th/0109165].
69. A. Vilenkin and E.P.S. Shellard, *Cosmic Strings and other Topological Defects* (Cambridge Univ. Press, Cambridge, 1994).
70. M. B. Hindmarsh and T. W. B. Kibble, "Cosmic strings," *Rept. Prog. Phys.* **58**, 477 (1995) [arXiv:hep-ph/9411342].
71. R. H. Brandenberger, "Topological defects and structure formation," *Int. J. Mod. Phys. A* **9**, 2117 (1994) [arXiv:astro-ph/9310041].
72. R. Easther, B. R. Greene, M. G. Jackson and D. Kabat, "String windings in the early universe," *JCAP* **0502**, 009 (2005) [arXiv:hep-th/0409121].
73. R. Danos, A. R. Frey and A. Mazumdar, "Interaction rates in string gas cosmology," *Phys. Rev. D* **70**, 106010 (2004) [arXiv:hep-th/0409162].
74. B. Greene, D. Kabat and S. Marnerides, "Dynamical Decompactification and Three Large Dimensions," *Phys. Rev. D* **82**, 043528 (2010) [arXiv:0908.0955 [hep-th]].
75. R. H. Brandenberger, "Moduli stabilization in string gas cosmology," *Prog. Theor. Phys. Suppl.* **163**, 358 (2006) [arXiv:hep-th/0509159].
76. S. Watson and R. Brandenberger, "Stabilization of extra dimensions at tree level," *JCAP* **0311**, 008 (2003) [arXiv:hep-th/0307044].
77. L. Kofman, A. Linde, X. Liu, A. Maloney, L. McAllister and E. Silverstein, "Beauty is attractive: Moduli trapping at enhanced symmetry points," *JHEP* **0405**, 030 (2004) [arXiv:hep-th/0403001].
78. S. Watson, "Moduli stabilization with the string Higgs effect," *Phys. Rev. D* **70**, 066005 (2004) [arXiv:hep-th/0404177].
79. S. P. Patil and R. Brandenberger, "Radion stabilization by stringy effects in general relativity and dilaton gravity," *Phys. Rev. D* **71**, 103522 (2005) [arXiv:hep-th/0401037].
80. S. P. Patil and R. H. Brandenberger, "The cosmology of massless string modes," *JCAP* **0601**, 005 (2006) [arXiv:hep-th/0502069].
81. R. Brandenberger, Y. K. Cheung and S. Watson, "Moduli stabilization with string gases and fluxes," *JHEP* **0605**, 025 (2006) [arXiv:hep-th/0501032].
82. R. J. Danos, A. R. Frey and R. H. Brandenberger, "Stabilizing moduli with thermal matter and nonperturbative effects," *Phys. Rev. D* **77**, 126009 (2008) [arXiv:0802.1557 [hep-th]].
83. C. P. Burgess and L. McAllister, "Challenges for String Cosmology," *Class. Quant. Grav.* **28**, 204002 (2011) [arXiv:1108.2660 [hep-th]].
84. S. Mishra, W. Xue, R. Brandenberger and U. Yajnik, "Supersymmetry Breaking and Dilaton Stabilization in String Gas Cosmology," arXiv:1103.1389 [hep-th].
85. S. Watson and R. H. Brandenberger, "Isotropization in brane gas cosmology," *Phys. Rev. D* **67**, 043510 (2003) [arXiv:hep-th/0207168].
86. V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, "Theory of cosmological perturbations. Part 1. Classical perturbations. Part 2. Quantum theory of perturbations. Part 3. Extensions," *Phys. Rept.* **215**, 203 (1992).
87. R. H. Brandenberger, "Lectures on the theory of cosmological perturbations," *Lect. Notes Phys.* **646**, 127 (2004) [arXiv:hep-th/0306071].
88. V. F. Mukhanov, "Quantum Theory of Gauge Invariant Cosmological Perturbations," *Sov. Phys. JETP* **67**, 1297 (1988) [*Zh. Eksp. Teor. Fiz.* **94N7**, 1 (1988)].
89. V. F. Mukhanov, "Gravitational Instability Of The Universe Filled With A Scalar Field," *JETP Lett.* **41**, 493 (1985) [*Pisma Zh. Eksp. Teor. Fiz.* **41**, 402 (1985)].

90. M. Sasaki, "Large Scale Quantum Fluctuations in the Inflationary Universe," *Prog. Theor. Phys.* **76**, 1036 (1986).
91. V. N. Lukash, "Production of phonons in an isotropic universe," *Sov. Phys. JETP* **52**, 807 (1980) [*Zh. Eksp. Teor. Fiz.* **79**, (1977)].
92. J. M. Bardeen, P. J. Steinhardt and M. S. Turner, "Spontaneous Creation Of Almost Scale - Free Density Perturbations In An Inflationary Universe," *Phys. Rev. D* **28**, 679 (1983).
93. R. H. Brandenberger and R. Kahn, "Cosmological Perturbations In Inflationary Universe Models," *Phys. Rev. D* **29**, 2172 (1984).
94. D. H. Lyth, "Large Scale Energy Density Perturbations And Inflation," *Phys. Rev. D* **31**, 1792 (1985).
95. N. D. Birrell and P. C. W. Davies, "Quantum Fields In Curved Space," *Cambridge, Uk: Univ. Pr. (1982) 340p*
96. C. Kiefer, I. Lohmar, D. Polarski and A. A. Starobinsky, "Pointer states for primordial fluctuations in inflationary cosmology," *Class. Quant. Grav.* **24**, 1699 (2007) [arXiv:astro-ph/0610700].
97. P. Martineau, "On the decoherence of primordial fluctuations during inflation," *Class. Quant. Grav.* **24**, 5817 (2007) [arXiv:astro-ph/0601134].
98. S. Alexander, T. Biswas and R. H. Brandenberger, "On the Transfer of Adiabatic Fluctuations through a Nonsingular Cosmological Bounce," arXiv:0707.4679 [hep-th].
99. X. Gao, Y. Wang, W. Xue and R. Brandenberger, "Fluctuations in a Hořava-Lifshitz Bouncing Cosmology," *JCAP* **1002**, 020 (2010) [arXiv:0911.3196 [hep-th]];
X. Gao, Y. Wang, R. Brandenberger and A. Riotto, "Cosmological Perturbations in Hořava-Lifshitz Gravity," *Phys. Rev. D* **81**, 083508 (2010) [arXiv:0905.3821 [hep-th]].
100. Y. F. Cai, W. Xue, R. Brandenberger and X. m. Zhang, "Thermal Fluctuations and Bouncing Cosmologies," *JCAP* **0906**, 037 (2009) [arXiv:0903.4938 [hep-th]].
101. J. c. Hwang and E. T. Vishniac, "Gauge-invariant joining conditions for cosmological perturbations," *Astrophys. J.* **382**, 363 (1991).
102. N. Deruelle and V. F. Mukhanov, "On matching conditions for cosmological perturbations," *Phys. Rev. D* **52**, 5549 (1995) [arXiv:gr-qc/9503050].
103. R. Durrer and F. Vernizzi, "Adiabatic perturbations in pre big bang models: Matching conditions and scale invariance," *Phys. Rev. D* **66**, 083503 (2002) [arXiv:hep-ph/0203275].
104. D. H. Lyth, "The Failure of cosmological perturbation theory in the new ekpyrotic scenario," *Phys. Lett. B* **526**, 173 (2002) [hep-ph/0110007];
D. H. Lyth, "The Primordial curvature perturbation in the ekpyrotic universe," *Phys. Lett. B* **524**, 1 (2002) [hep-ph/0106153].
105. R. Brandenberger and F. Finelli, "On the spectrum of fluctuations in an effective field theory of the Ekpyrotic universe," *JHEP* **0111**, 056 (2001) [hep-th/0109004].
106. J. Khoury, B. A. Ovrut, P. J. Steinhardt and N. Turok, "Density perturbations in the ekpyrotic scenario," *Phys. Rev. D* **66**, 046005 (2002) [hep-th/0109050].
107. R. H. Brandenberger, "Processing of Cosmological Perturbations in a Cyclic Cosmology," *Phys. Rev. D* **80**, 023535 (2009) [arXiv:0905.1514 [hep-th]].
108. P. J. Steinhardt and N. Turok, "Cosmic evolution in a cyclic universe," *Phys. Rev. D* **65**, 126003 (2002) [hep-th/0111098].
109. Y. Cai, R. Brandenberger and X. Zhang, "The Matter Bounce Curvaton Scenario", *JCAP* **1103**, 003 (2011) [arXiv:1101.0822 [hep-th]].
110. A. Cerioni and R. H. Brandenberger, "Cosmological Perturbations in the Projectable Version of Horava-Lifshitz Gravity," *JCAP* **1108**, 015 (2011) [arXiv:1007.1006 [hep-th]].
111. A. Cerioni and R. H. Brandenberger, "Cosmological Perturbations in the 'Healthy Extension' of Horava-Lifshitz gravity," arXiv:1008.3589 [hep-th].
112. Y. F. Cai, W. Xue, R. Brandenberger and X. Zhang, "Non-Gaussianity in a Matter Bounce," *JCAP* **0905**, 011 (2009) [arXiv:0903.0631 [astro-ph.CO]].
113. X. Chen, "Primordial Non-Gaussianities from Inflation Models," *Adv. Astron.* **2010**, 638979 (2010) [arXiv:1002.1416 [astro-ph.CO]].
114. J. M. Maldacena, "Non-Gaussian features of primordial fluctuations in single field inflationary models," *JHEP* **0305**, 013 (2003) [arXiv:astro-ph/0210603].

115. H. Li, J. Q. Xia, R. Brandenberger and X. Zhang, “Constraints on Models with a Break in the Primordial Power Spectrum,” *Phys. Lett. B* **690**, 451 (2010) [arXiv:0903.3725 [astro-ph.CO]].
116. R. H. Brandenberger, A. Nayeri, S. P. Patil and C. Vafa, “String gas cosmology and structure formation,” *Int. J. Mod. Phys. A* **22**, 3621 (2007) [arXiv:hep-th/0608121].
117. R. H. Brandenberger, A. Nayeri, S. P. Patil and C. Vafa, “Tensor modes from a primordial Hagedorn phase of string cosmology,” *Phys. Rev. Lett.* **98**, 231302 (2007) [arXiv:hep-th/0604126].
118. R. H. Brandenberger *et al.*, “More on the spectrum of perturbations in string gas cosmology,” *JCAP* **0611**, 009 (2006) [arXiv:hep-th/0608186].
119. N. Kaloper, L. Kofman, A. Linde and V. Mukhanov, “On the new string theory inspired mechanism of generation of cosmological perturbations,” *JCAP* **0610**, 006 (2006) [arXiv:hep-th/0608200].
120. N. Deo, S. Jain, O. Narayan and C. I. Tan, “The Effect of topology on the thermodynamic limit for a string gas,” *Phys. Rev. D* **45**, 3641 (1992).
121. A. Nayeri, “Inflation free, stringy generation of scale-invariant cosmological fluctuations in $D = 3 + 1$ dimensions,” arXiv:hep-th/0607073.
122. L. P. Grishchuk, “Amplification Of Gravitational Waves In An Istropic Universe,” *Sov. Phys. JETP* **40**, 409 (1975) [*Zh. Eksp. Teor. Fiz.* **67**, 825 (1974)].
123. B. Chen, Y. Wang, W. Xue and R. Brandenberger, “String Gas Cosmology and Non-Gaussianities,” arXiv:0712.2477 [hep-th].
124. E. Witten, “Cosmic Superstrings,” *Phys. Lett. B* **153**, 243 (1985);
E. J. Copeland, R. C. Myers and J. Polchinski, “Cosmic F and D strings,” *JHEP* **0406**, 013 (2004) [hep-th/0312067].
125. N. Kaiser and A. Stebbins, “Microwave Anisotropy Due To Cosmic Strings,” *Nature* **310**, 391 (1984).
126. S. Amsel, J. Berger and R. H. Brandenberger, “Detecting Cosmic Strings in the CMB with the Canny Algorithm,” *JCAP* **0804**, 015 (2008) [arXiv:0709.0982 [astro-ph]];
A. Stewart and R. Brandenberger, “Edge Detection, Cosmic Strings and the South Pole Telescope,” *JCAP* **0902**, 009 (2009) [arXiv:0809.0865 [astro-ph]];
R. J. Danos and R. H. Brandenberger, “Canny Algorithm, Cosmic Strings and the Cosmic Microwave Background,” *Int. J. Mod. Phys. D* **19**, 183 (2010) [arXiv:0811.2004 [astro-ph]];
R. J. Danos and R. H. Brandenberger, “Searching for Signatures of Cosmic Superstrings in the CMB,” *JCAP* **1002**, 033 (2010) [arXiv:0910.5722 [astro-ph.CO]].